

## Equation de Convection-Diffusion 1D

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### EXEMPLE 1

Détermination de la distribution de  $\phi(x)$  transportée par convection-diffusion à travers un domaine 1D dont les extrémités sont soumises aux (C.L.):  
Schéma Upwind pour le terme convectif.

$$\frac{d}{dx} (\rho \cdot u \cdot \phi(x)) - \frac{d}{dx} \left( \Gamma \cdot \frac{d}{dx} \phi(x) \right) = 0$$

Conditions aux limites (C.L.):

$$\phi(0) = \phi_0 = 1,$$

$$\phi(L) = \phi_L = 0,$$

$u = 0.1$  m/s ,  $\nu = 0.2$ :

> *Restart : Digits := 4 :*

>  $u := 0.1$ ;  $L := 1.0$ ;  $\rho := 1.0$ ;  $\text{Gam} := 0.1$ ;  $S := 1.0$ ;  $ndx := 5$ ;

$u := 0.1$

$L := 1.0$

$\rho := 1.0$

$\text{Gam} := 0.1$

$S := 1.0$

$ndx := 5$

>  $\delta x := \frac{L}{ndx}$  ;

(1.1)

$$F := \rho \cdot u \cdot S;$$

$$d := \frac{Gam \cdot S}{\delta x};$$

$$Pe := \frac{F}{d};$$

$$\delta x := 0.2000$$

$$F := 0.100$$

$$d := 0.5000$$

$$Pe := 0.2000$$

(1.2)

$$> i_{\max} := ndx;$$

$$i_{\max} := 5$$

(1.3)

Nombre d'équations:

$$> Ne := i_{\max}$$

$$Ne := 5$$

(1.4)

Abscisses des noeuds:

$$> x[0] := 0;$$

**for**  $i$  **from** 1 **to**  $i_{\max}$  **do**

$$x[i] := \frac{\delta x}{2} + (i - 1) \cdot \delta x;$$

**end do;**

$$x[i_{\max} + 1] := L;$$

$$x_0 := 0$$

$$x_1 := 0.1000$$

$$x_2 := 0.3000$$

$$x_3 := 0.5000$$

$$x_4 := 0.7000$$

$$x_5 := 0.9000$$

$$x_6 := 1.0$$

(1.5)

Conditions aux Limites:

$$> \phi[0] := 1.0;$$

$$\phi[i_{\max} + 1] := 0;$$

$$\phi[L] := \phi[i_{\max} + 1];$$

$$\phi_0 := 1.0$$

$$\phi_6 := 0$$

$$\phi_{1.0} := 0$$

(1.6)

Noeuds internes:

**> for**  $i$  **from** 2 **to**  $i_{\max} - 1$  **do**

$$Sp[i] := 0;$$

$$Su[i] := 0;$$

$$a_w[i] := d + F;$$

$$a_E[i] := d;$$

$$a_P[i] := a_w[i] + a_E[i] - Sp[i];$$

**end do;**

$$Sp_2 := 0$$

$$Su_2 := 0$$

$$\begin{aligned}
a_{W_2} &:= 0.6000 \\
a_{E_2} &:= 0.5000 \\
a_{P_2} &:= 1.100 \\
Sp_3 &:= 0 \\
Su_3 &:= 0 \\
a_{W_3} &:= 0.6000 \\
a_{E_3} &:= 0.5000 \\
a_{P_3} &:= 1.100 \\
Sp_4 &:= 0 \\
Su_4 &:= 0 \\
a_{W_4} &:= 0.6000 \\
a_{E_4} &:= 0.5000 \\
a_{P_4} &:= 1.100
\end{aligned} \tag{1.7}$$

Noeud gauche:

$$\begin{aligned}
> Sp[1] &:= - (2 \cdot d + F); \\
Su[1] &:= (2 \cdot d + F) \cdot \phi[0]; \\
a_W[1] &:= 0; \\
a_E[1] &:= d; \\
a_P[1] &:= a_W[1] + a_E[1] - Sp[1]; \\
Sp_1 &:= -1.100 \\
Su_1 &:= 1.100 \\
a_{W_1} &:= 0 \\
a_{E_1} &:= 0.5000 \\
a_{P_1} &:= 1.600
\end{aligned} \tag{1.8}$$

Noeud droit:

$$\begin{aligned}
> Sp[i_{\max}] &:= - 2 \cdot d; \\
Su[i_{\max}] &:= 2 \cdot d \cdot \phi[i_{\max} + 1]; \\
a_W[i_{\max}] &:= d + F; \\
a_E[i_{\max}] &:= 0; \\
a_P[i_{\max}] &:= a_W[i_{\max}] + a_E[i_{\max}] - Sp[i_{\max}]; \\
Sp_5 &:= -1.000 \\
Su_5 &:= 0. \\
a_{W_5} &:= 0.6000 \\
a_{E_5} &:= 0 \\
a_{P_5} &:= 1.600
\end{aligned} \tag{1.9}$$

Equations:

$$\begin{aligned}
> k &:= 1 \\
k &:= 1
\end{aligned} \tag{1.1.1}$$

Résolution pour les noeuds internes:

> for i from 1 to Ne do

$$Eq[k] := a_P[i] \cdot \phi[i] = a_W[i] \cdot \phi[i-1] + a_E[i] \cdot \phi[i+1] + Su[i];$$

$$k := k + 1;$$

end do;

$$Eq_1 := 1.600 \phi_1 = 1.100 + 0.5000 \phi_2$$

$$k := 2$$

$$Eq_2 := 1.100 \phi_2 = 0.6000 \phi_1 + 0.5000 \phi_3$$

$$k := 3$$

$$Eq_3 := 1.100 \phi_3 = 0.6000 \phi_2 + 0.5000 \phi_4$$

$$k := 4$$

$$Eq_4 := 1.100 \phi_4 = 0.6000 \phi_3 + 0.5000 \phi_5$$

$$k := 5$$

$$Eq_5 := 1.600 \phi_5 = 0.6000 \phi_4$$

$$k := 6$$

(1.1.2)

Ecriture du système d'équations:

> for k from 1 to Ne do Eq[k] end do;

$$1.600 \phi_1 = 1.100 + 0.5000 \phi_2$$

$$1.100 \phi_2 = 0.6000 \phi_1 + 0.5000 \phi_3$$

$$1.100 \phi_3 = 0.6000 \phi_2 + 0.5000 \phi_4$$

$$1.100 \phi_4 = 0.6000 \phi_3 + 0.5000 \phi_5$$

$$1.600 \phi_5 = 0.6000 \phi_4$$

(1.1.3)

> Eqs := {seq(Eq[k], k=1..Ne)};

$$Eqs := \{1.600 \phi_1 = 1.100 + 0.5000 \phi_2, 1.100 \phi_2 = 0.6000 \phi_1 + 0.5000 \phi_3, 1.100 \phi_3 = 0.6000 \phi_2 + 0.5000 \phi_4, 1.100 \phi_4 = 0.6000 \phi_3 + 0.5000 \phi_5, 1.600 \phi_5 = 0.6000 \phi_4\} \quad (1.1.4)$$

> Tmps := [seq(\phi[i], i=1..Ne)];

$$Tmps := [\phi_1, \phi_2, \phi_3, \phi_4, \phi_5]$$

(1.1.5)

> SolT := solve(Eqs, Tmps);

$$SolT := [[\phi_1 = 0.9337, \phi_2 = 0.7879, \phi_3 = 0.6130, \phi_4 = 0.4031, \phi_5 = 0.1512]] \quad (1.1.6)$$

> with(LinearAlgebra) :

Forme matricielle:

> A, b := GenerateMatrix(Eqs, Tmps)

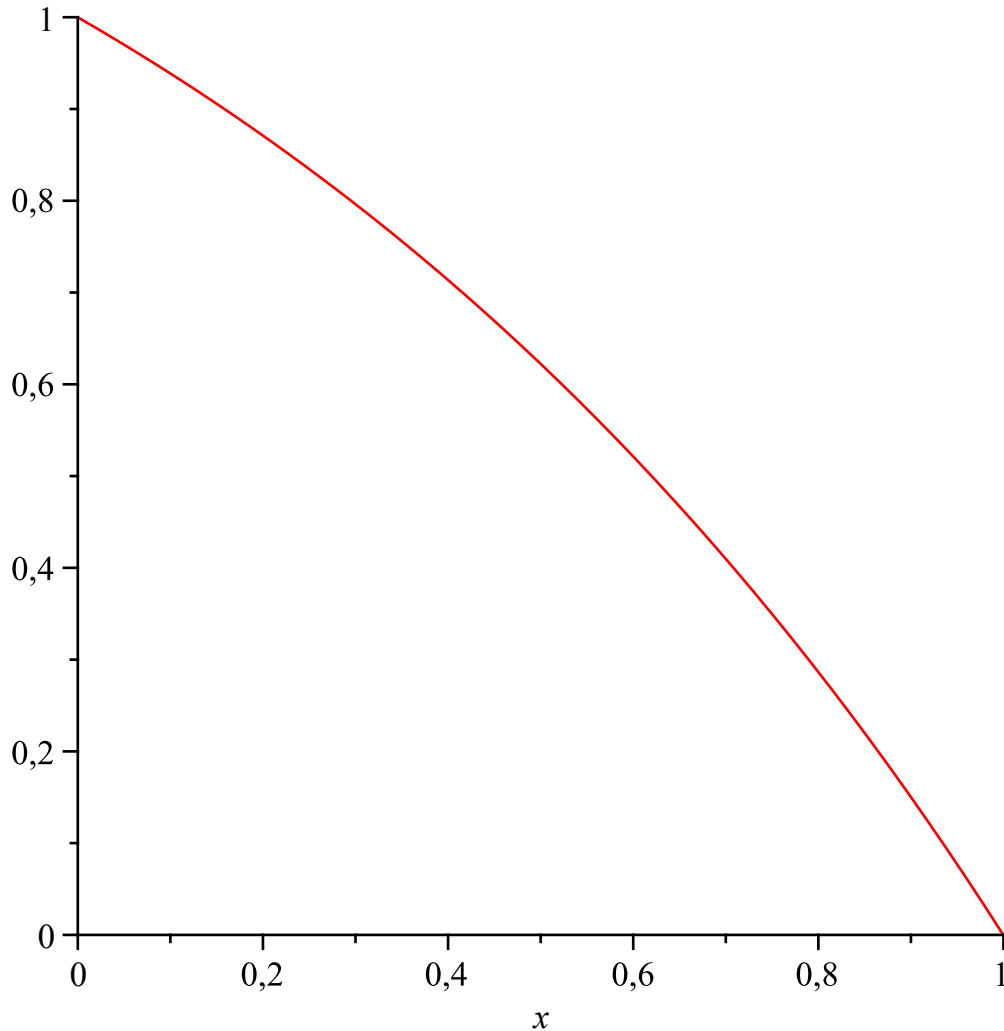
$$A, b := \begin{bmatrix} 1.600 & -0.5000 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.6000 & 1.600 \\ -0.6000 & 1.100 & -0.5000 & 0 & 0 \\ 0 & -0.6000 & 1.100 & -0.5000 & 0 \\ 0 & 0 & -0.6000 & 1.100 & -0.5000 \end{bmatrix}, \begin{bmatrix} 1.100 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (1.1.7)$$

Solution exacte:

$$> F(x) := \phi[0] + (\phi[L] - \phi[0]) \cdot \frac{e^{\left(\frac{\rho \cdot u \cdot x}{Gam}\right)} - 1}{e^{\left(\frac{\rho \cdot u \cdot L}{Gam}\right)} - 1};$$

$$F := x \rightarrow \phi_0 + \frac{(\phi_L - \phi_0) \left( e^{\frac{\rho u x}{Gam}} - 1 \right)}{e^{\frac{\rho u L}{Gam}} - 1} \quad (1.1.8)$$

```
> with(plots) :
> plot(F(x), x=0..L);
```



```
> for i from 1 to Ne do
     $\phi[i] := rhs(SolT_{1,i})$ 
end do;
```

```
 $\phi_1 := 0.9337$ 
 $\phi_2 := 0.7879$ 
 $\phi_3 := 0.6130$ 
 $\phi_4 := 0.4031$ 
 $\phi_5 := 0.1512$ 
```

(1.1.9)

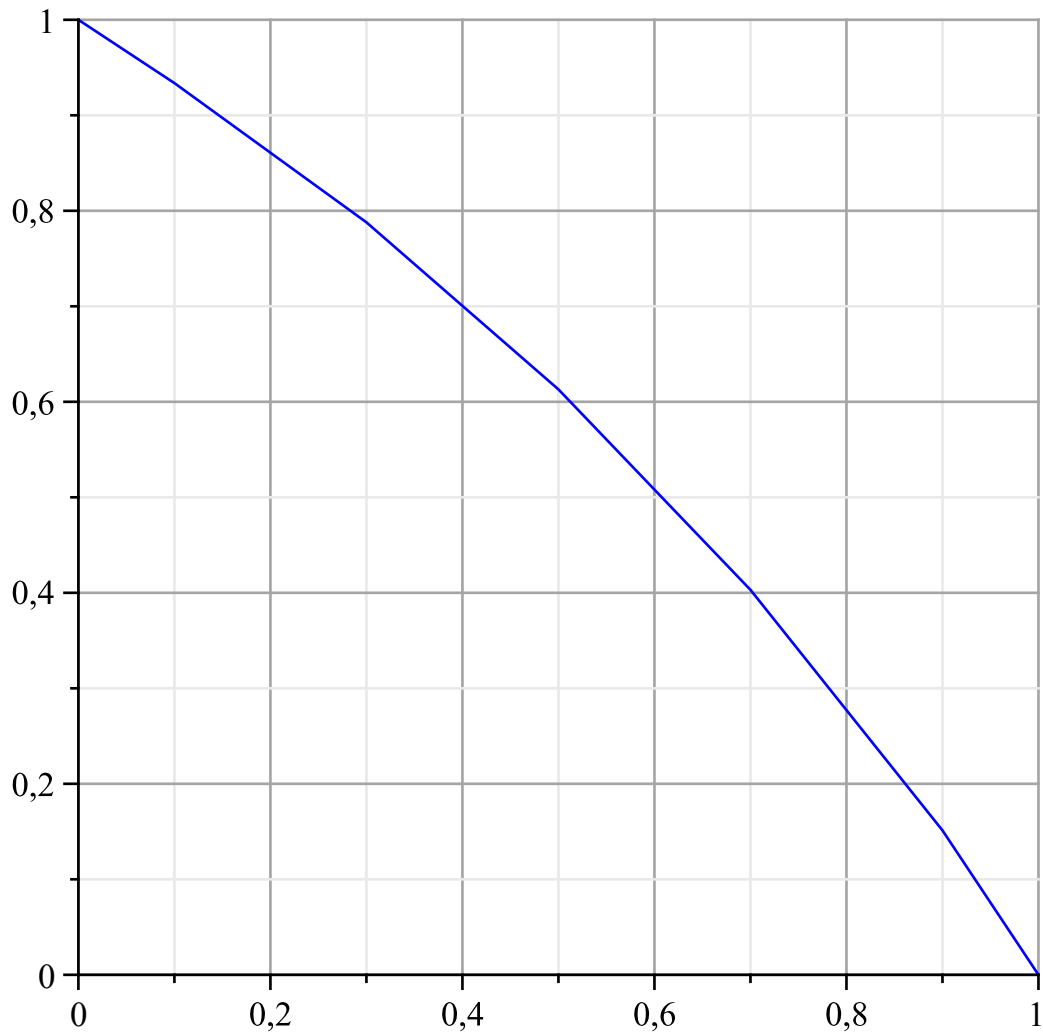
```
> lpN := [ seq( [x[i],  $\phi[i]$ ], i=0..i_max + 1 ) ]
```

```
lpN := [[0, 1.0], [0.1000, 0.9337], [0.3000, 0.7879], [0.5000, 0.6130], [0.7000, 0.4031], [0.9000, 0.1512], [1.0, 0]]
```

(1.1.10)

Courbe Numérique:

```
> listplot(lpN, color = blue, gridlines = true)
```



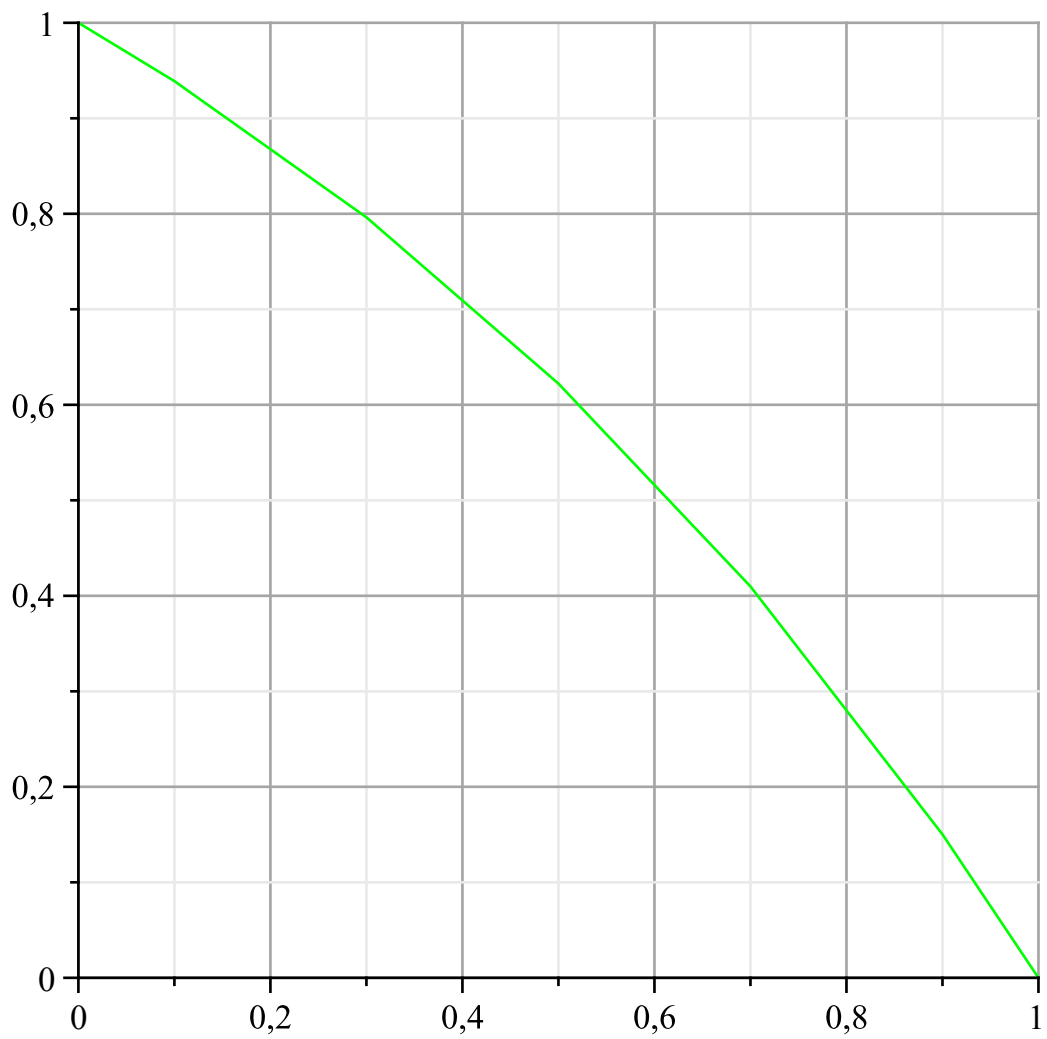
```
> lpT := [seq( [x[i], F(x[i]) ], i = 0 .. i_max + 1 ) ]
```

```
lpT := [ [0, 1.0], [0.1000, 0.9389], [0.3000, 0.7963], [0.5000, 0.6222], [0.7000, 0.4098], [0.9000, 0.1502], [1.0, 0.] ]
```

**(1.1.11)**

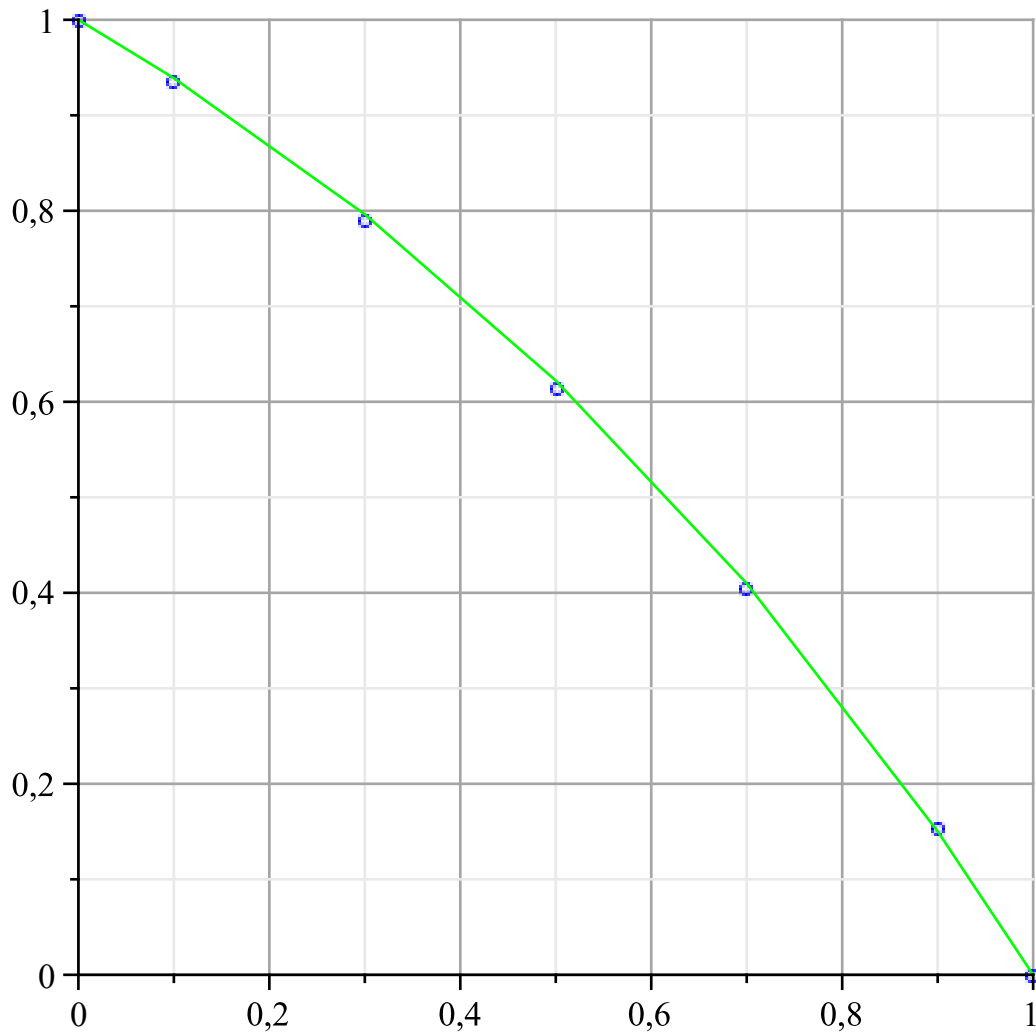
Courbe Théorique avec une liste de points:

```
> listplot(lpT, color = green, gridlines = true)
```



Tracé des deux courbes ensemble:

```
> multiple(listplot, [lpN, color = blue, style = point, symbol = circle], [lpT, color = green, style = line], color = black, gridlines = true)
```



Erreur relative:

```

> for i from 1 to Ne do
  x[i];
  phi[i];
  F(x[i]);
  (phi[i] - F(x[i])) / F(x[i]) * 100
end do

```

```

0.1000
0.9337
0.9389
-0.5538
0.3000
0.7879
0.7963
-1.055
0.5000
0.6130
0.6222
-1.479
0.7000
0.4031
0.4098
-1.635

```



0.9000  
0.1512  
0.1502  
0.6658

(1.1.12)

```
> P1 := plot(F(x), x=0 .. L, labels = [x, T], legend = Solution exacte) :  
> P2 := plot(lpN, color = blue, style = point, legend = Solution numérique) :  
display({P1, P2}, gridlines = true);
```

