$$\ddot{y} + 2\beta\dot{y} + \omega_0^2 y = f(t)$$

Recherchons la solution homogene

$$f(t) = 0$$

Equation homogène :

$$\ddot{y} + 2\beta\dot{y} + \omega_0^2 y = 0$$

Equation caractéristique :

$$r^2 + 2\beta r + \omega_0^2 = 0$$

Δ>0

$$y_h(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$

Δ=0

$$y_h(t) = (C_1 + C_2 t)e^{rt}$$

Δ<0

$$y_h(t) = Ae^{-\beta t}\cos(wt + \varphi)$$

$$w = \sqrt{{w_0}^2 - \beta^2}$$

Recherchons la solution particuliere

$$f(t) \neq 0$$

Equation générale :

$$\ddot{y} + 2\beta\dot{y} + \omega_0^2 y = f(t)$$

$$f(t) = A_0$$

$$y_p(t) = \frac{A_0}{\omega_0^2}$$

$$f(t) = A_0\cos(\Omega t)$$

$$y_p(t) = y_0 \cos(\Omega t + \varphi)$$

$$y_0 = \frac{A_0}{\sqrt{(w_0^2 - \Omega^2)^2 + 4\beta^2\Omega^2}}$$

$$\varphi = -arctg\left(\frac{2\beta\Omega}{w_0^2 - \Omega^2}\right)$$