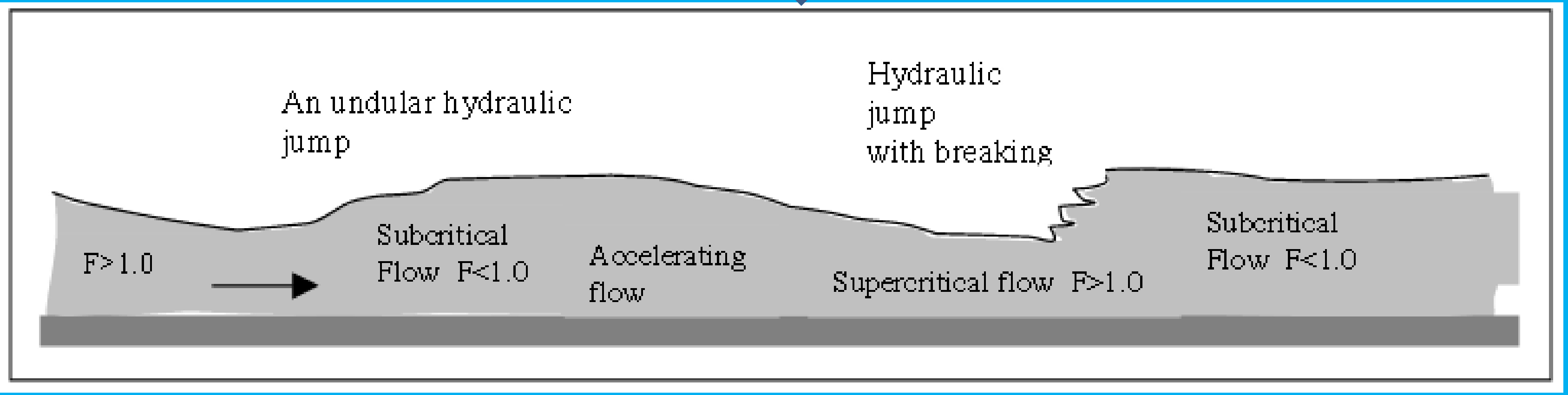
Steady Flow

Non Uniform Flow

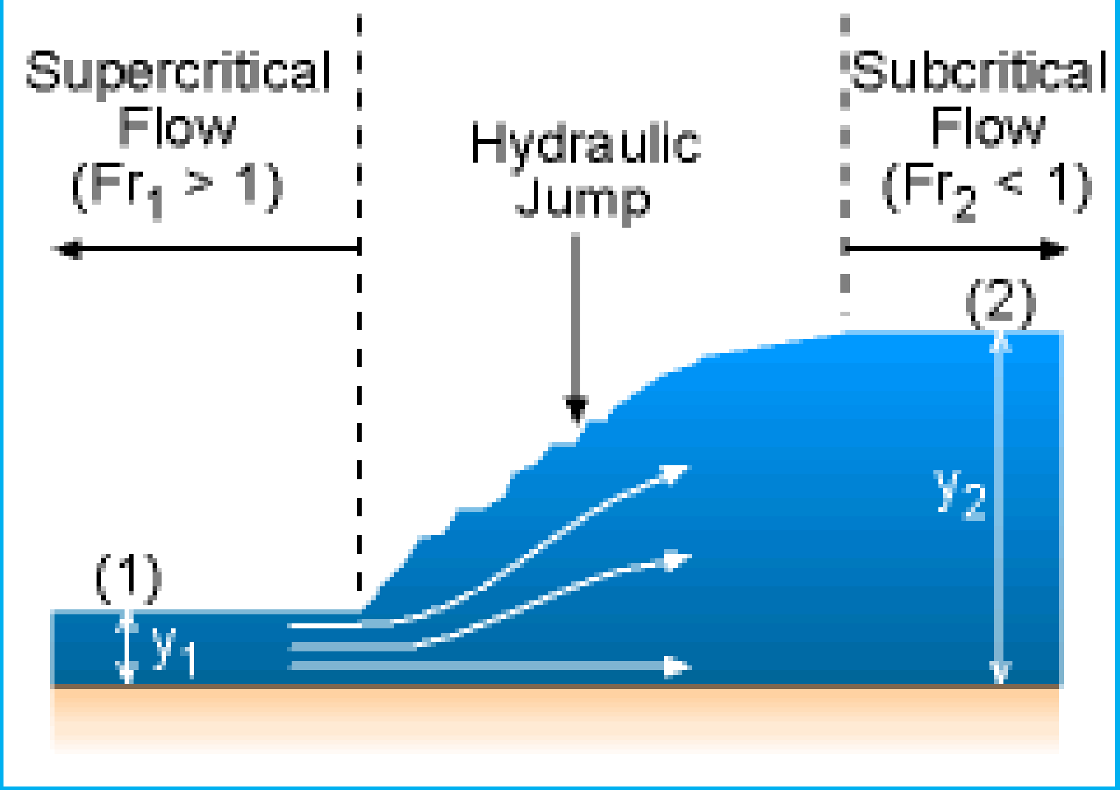
Rapidly varied flow

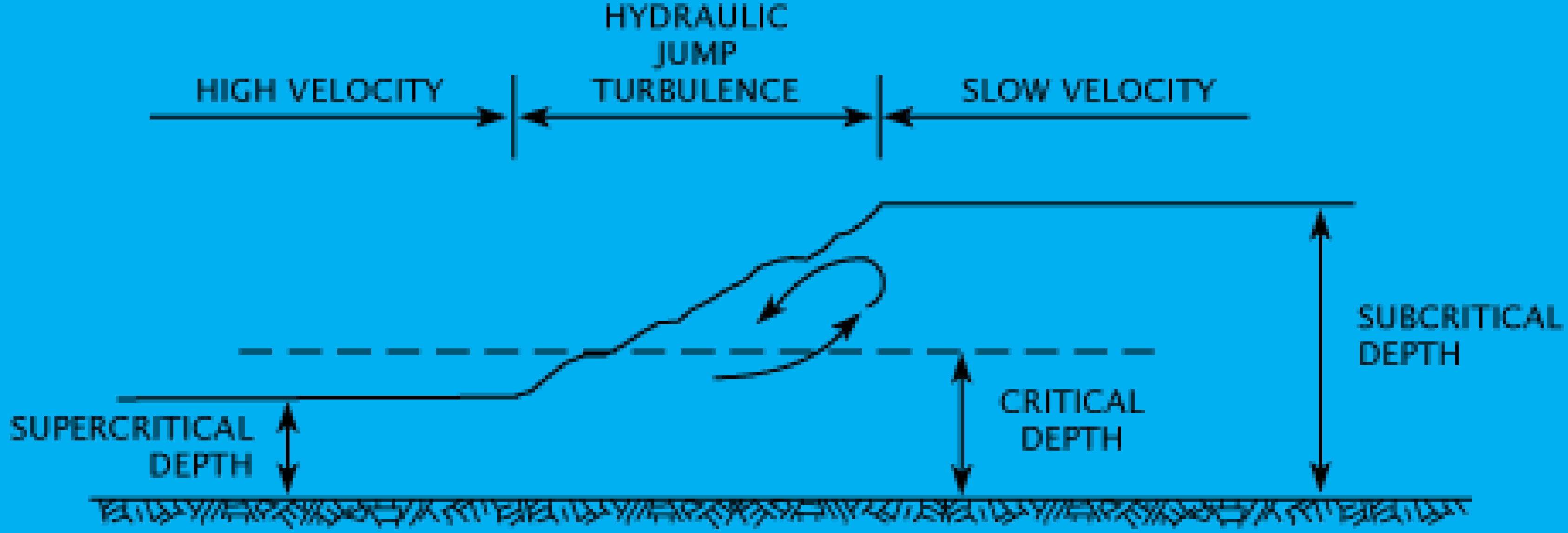
Hydraulic jump

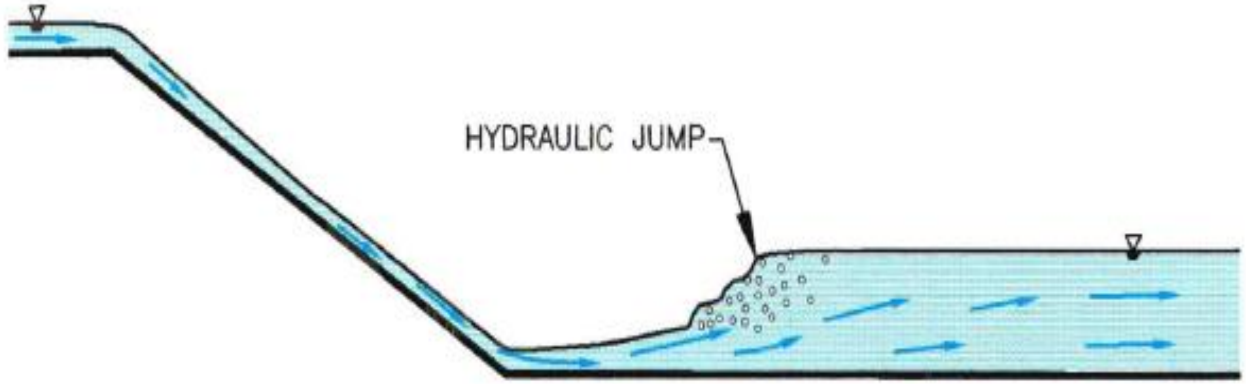
Hydraulic jump



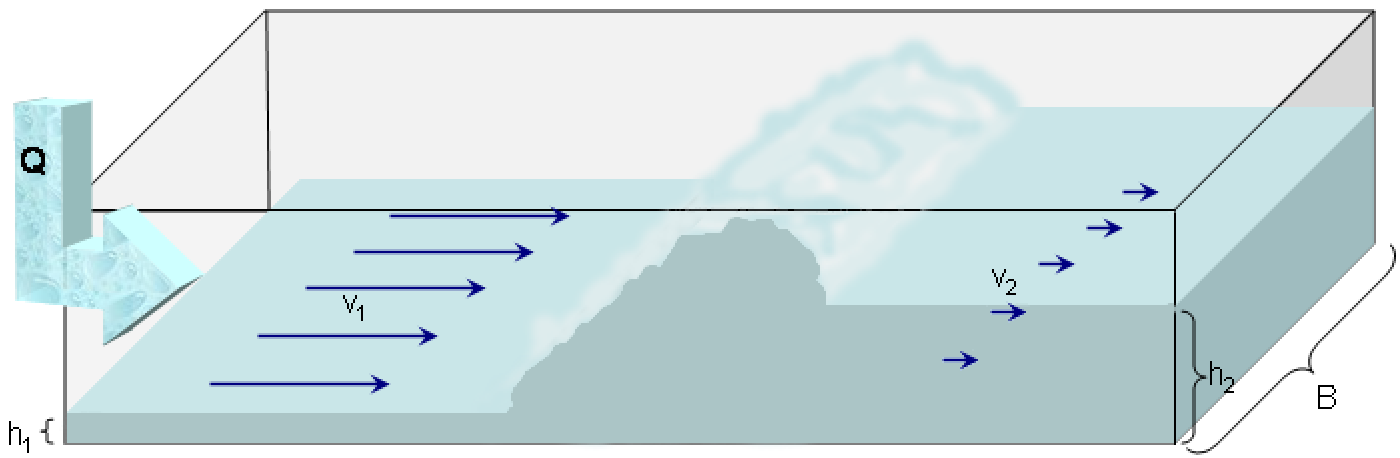
Undular and breaking hydraulic jumps in open-channel flow


Supercritical Flow ($Fr_1 > 1$)
Hydraulic Jump
Subcritical Flow ($Fr_2 < 1$)
(2)
(1)
y_1
y_2

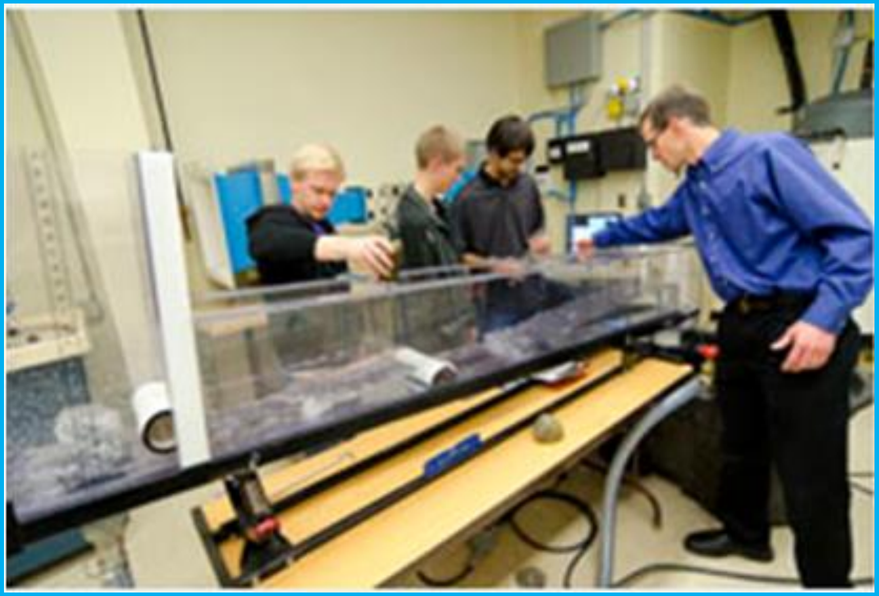

HYDRAULIC JUMP TURBULENCE
HIGH VELOCITY
SLOW VELOCITY
SUBCRITICAL DEPTH
CRITICAL DEPTH
SUPERCRITICAL DEPTH

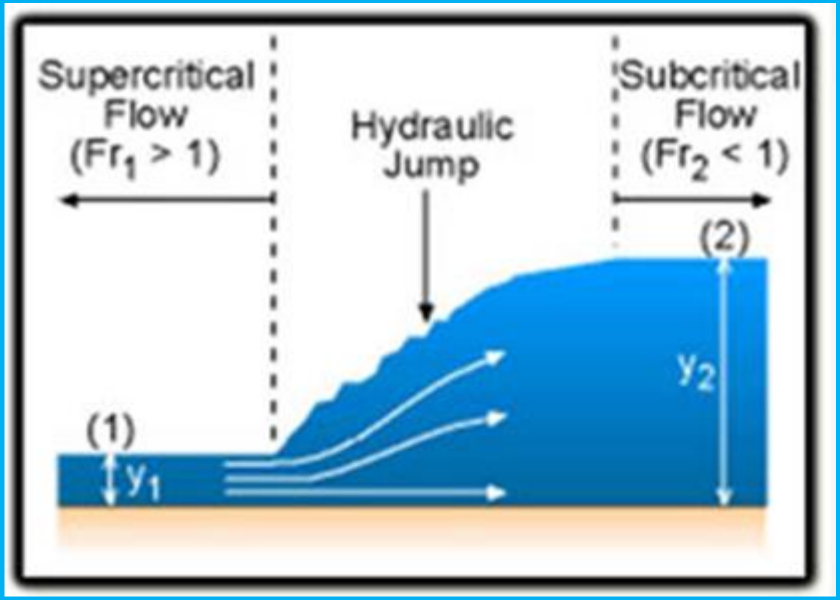


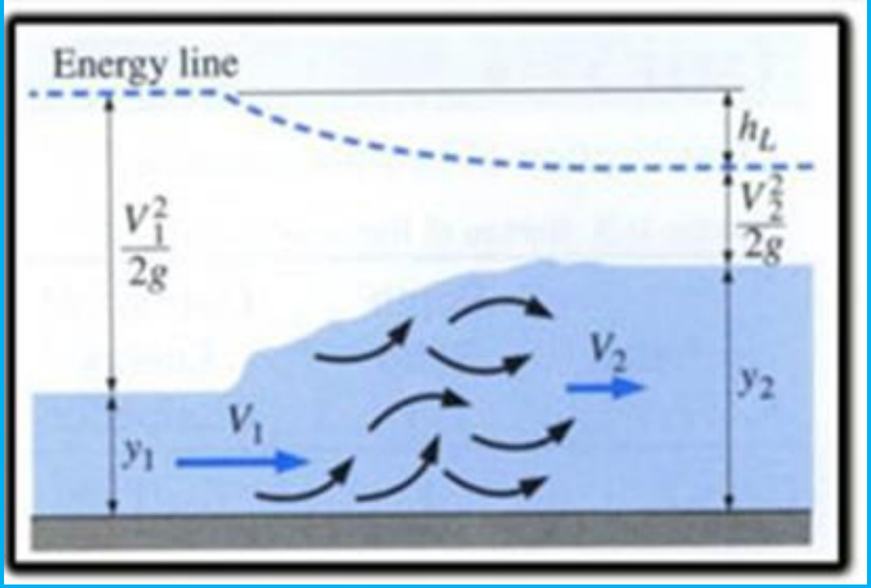
Courtesy of Wright Water Engineers, Inc. and ASDSO.
Source: Wright, Kenneth R., Kelly, Jonathan M., Houghtalen, Robert J., & Bonner, Mark R. "Emergency Rescues at Low-Head Dams." Paper presented at Dam Safety 1995, the 12th annual conference of the Association of State Dam Safety Officials, Atlanta, GA, September 1995.

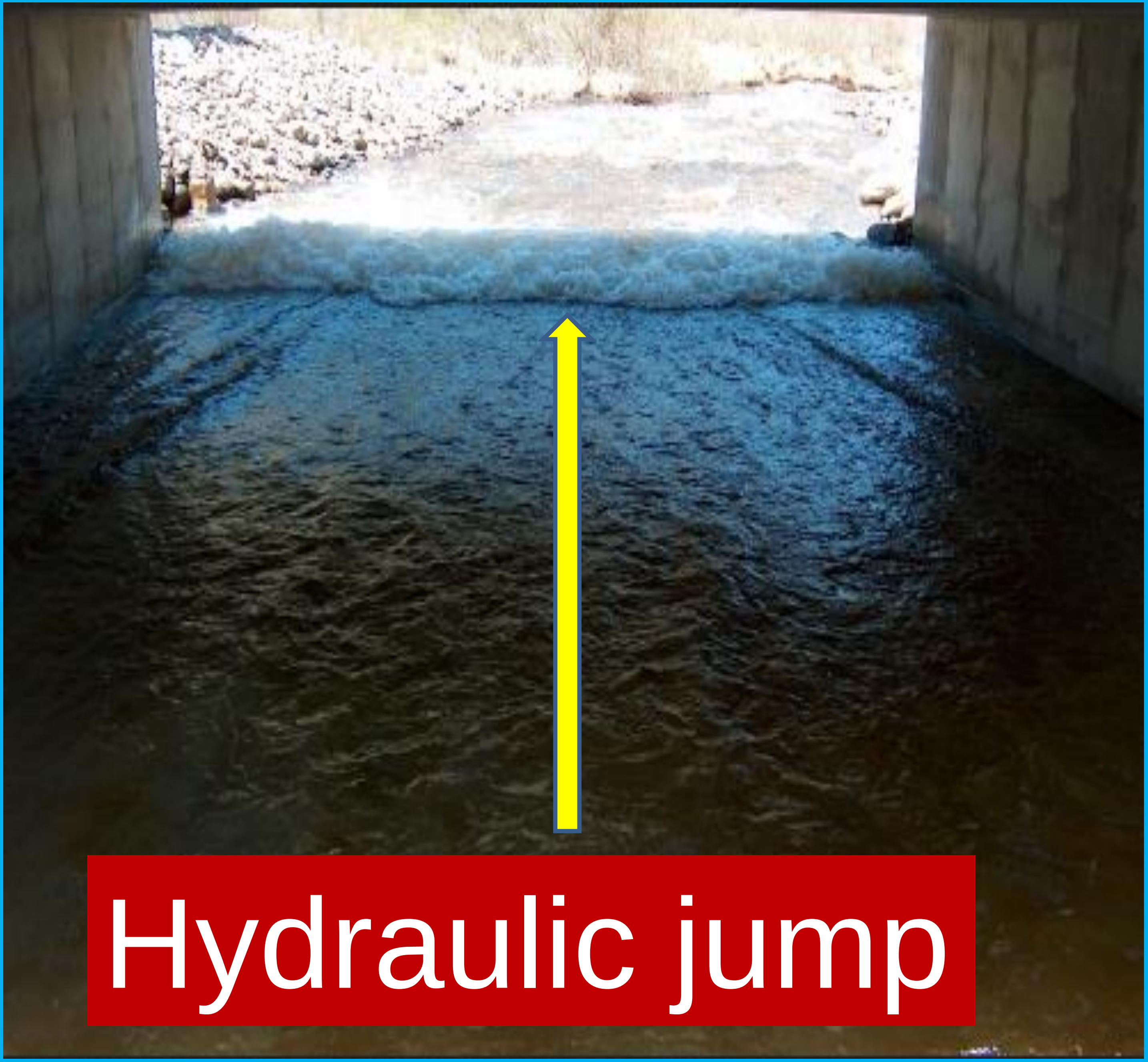


Q: flow rate
B: channel width
h_1: upstream depth
v_1: upstream velocity
h_2: downstream depth
v_2: downstream velocity




Supercritical Flow (Fr1 > 1)
Hydraulic Jump
Subcritical Flow (Fr2 < 1)
(1)
(2)
y1
y2


Energy line
hL
V1²/2g
V2²/2g
V1
V2
y1
y2


Hydraulic jump

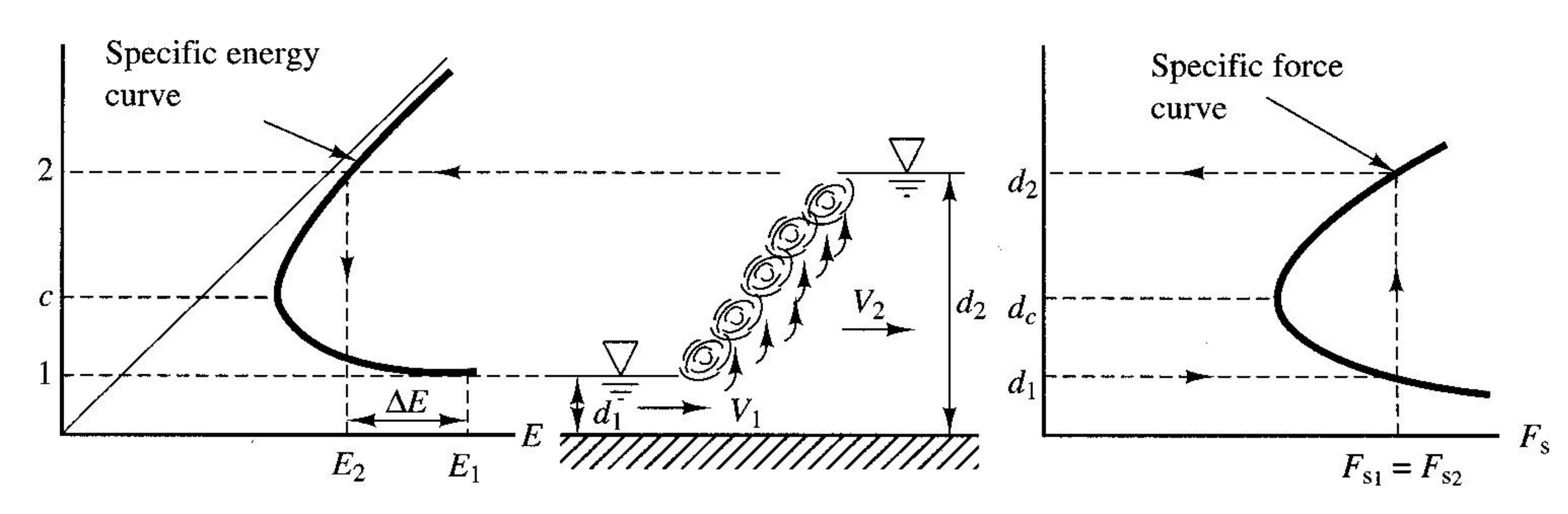
Hydraulic Jump

1.A hydraulic jump occurs when flow changes from a supercritical flow (unstable) to a sub-critical flow (stable).

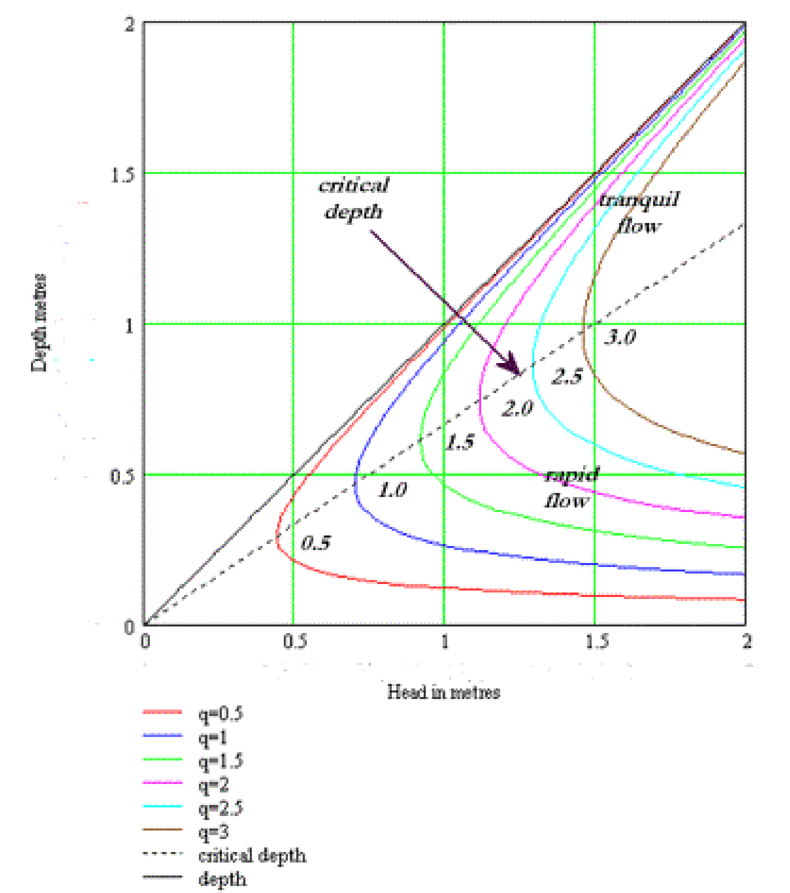
2. There is a sudden rise in water level at the point where the hydraulic jump occurs.

3. Rollers (eddies) of turbulent water form at this point. These rollers cause dissipation of energy.

4. A hydraulic jump occurs in practice at the toe of a dam or below a sluice gate where the velocity is very high.



Specific energy head diagram for q =const



Graph 10-3

General Expression for Hydraulic Jump:

In the analysis of hydraulic jumps, the following assumptions are made:

(1) **The length of hydraulic jump is small**. Consequently, the loss of head due to friction is negligible.

(2) The flow is uniform and pressure distribution is due to hydrostatic before and after the jump.

(3) The slope of the bed of the channel is very small, so that the component of the weight of the fluid in the direction of the flow is neglected.

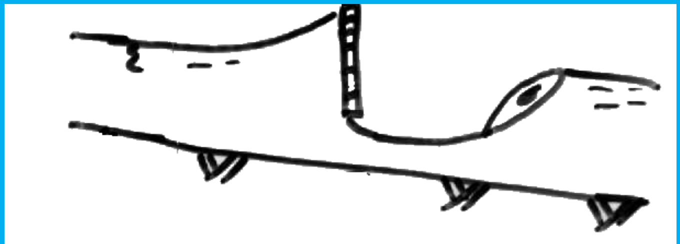
Location of hydraulic jump

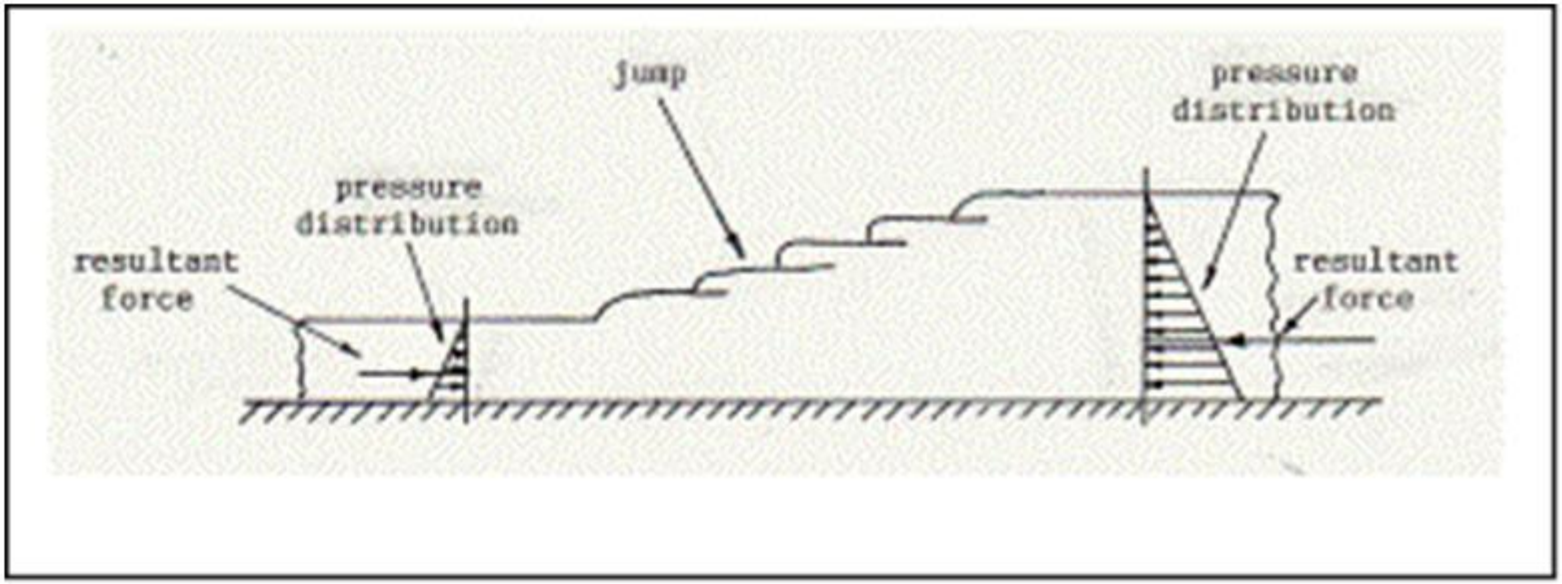
Generally, a hydraulic jump occurs when the flow changes from supercritical to subcritical flow.

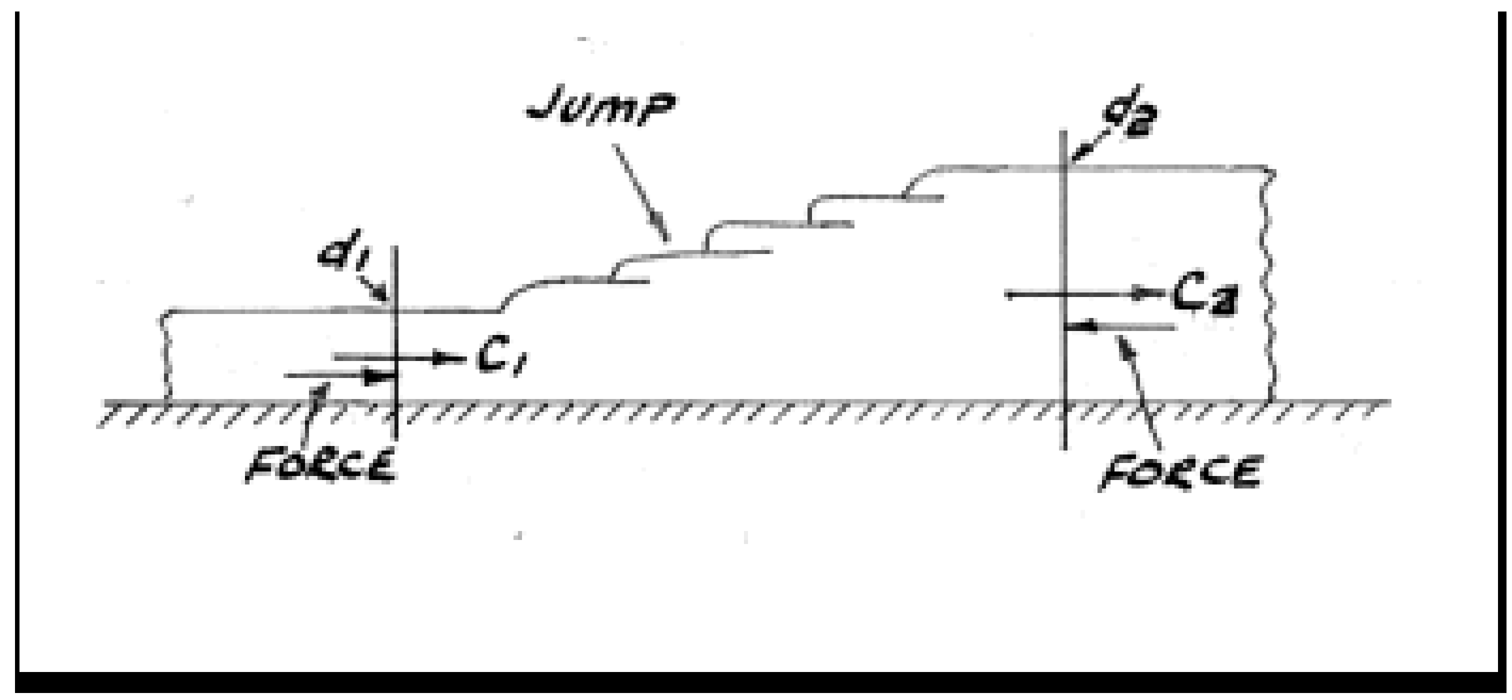
The most typical cases for the location of hydraulic jump are:

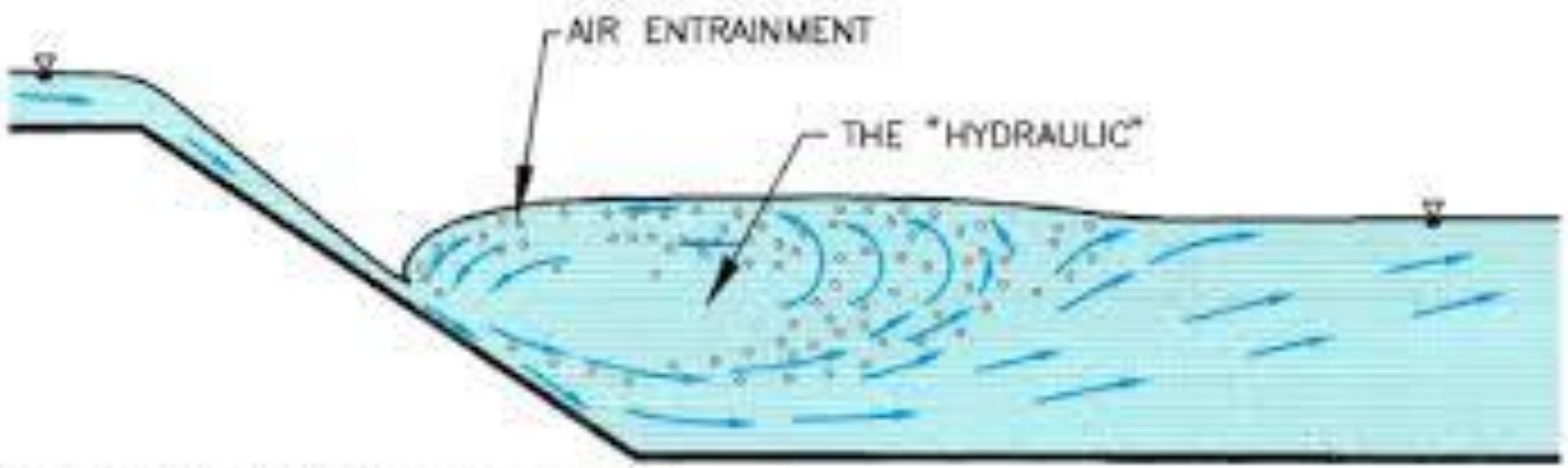
1. Jump below a sluice gate.
2. Jump at the toe of a spillway.
3. Jump at a glacis.

(*glacis* is the name given to sloping floors provided in hydraulic structures.)




jump
pressure distribution
pressure distribution
resultant force
resultant force


JUMP
d_2
d_1
C_1
C_2
FORCE
FORCE



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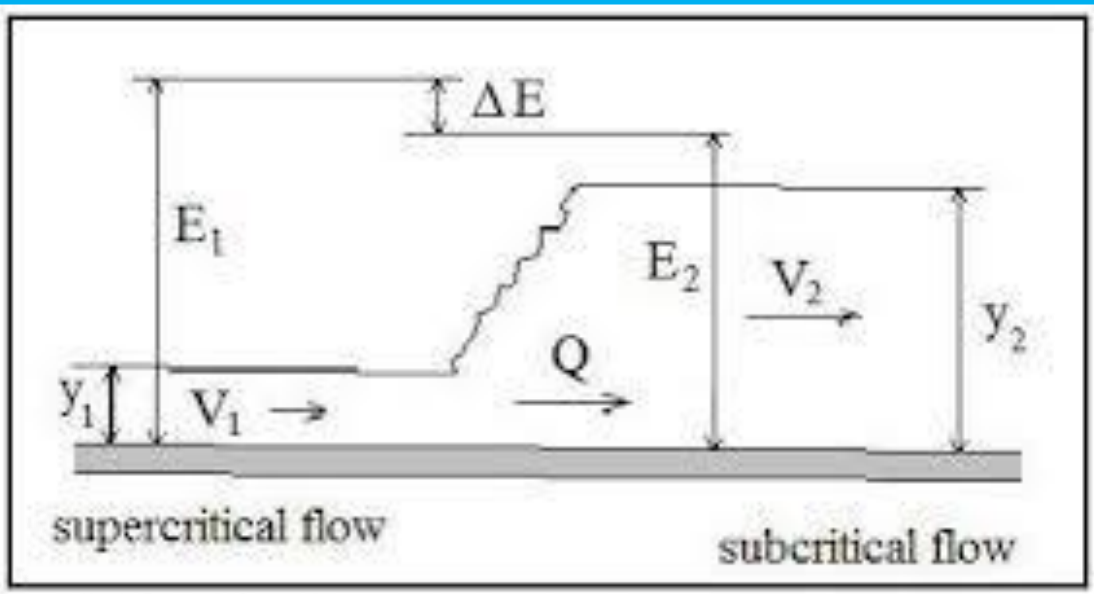
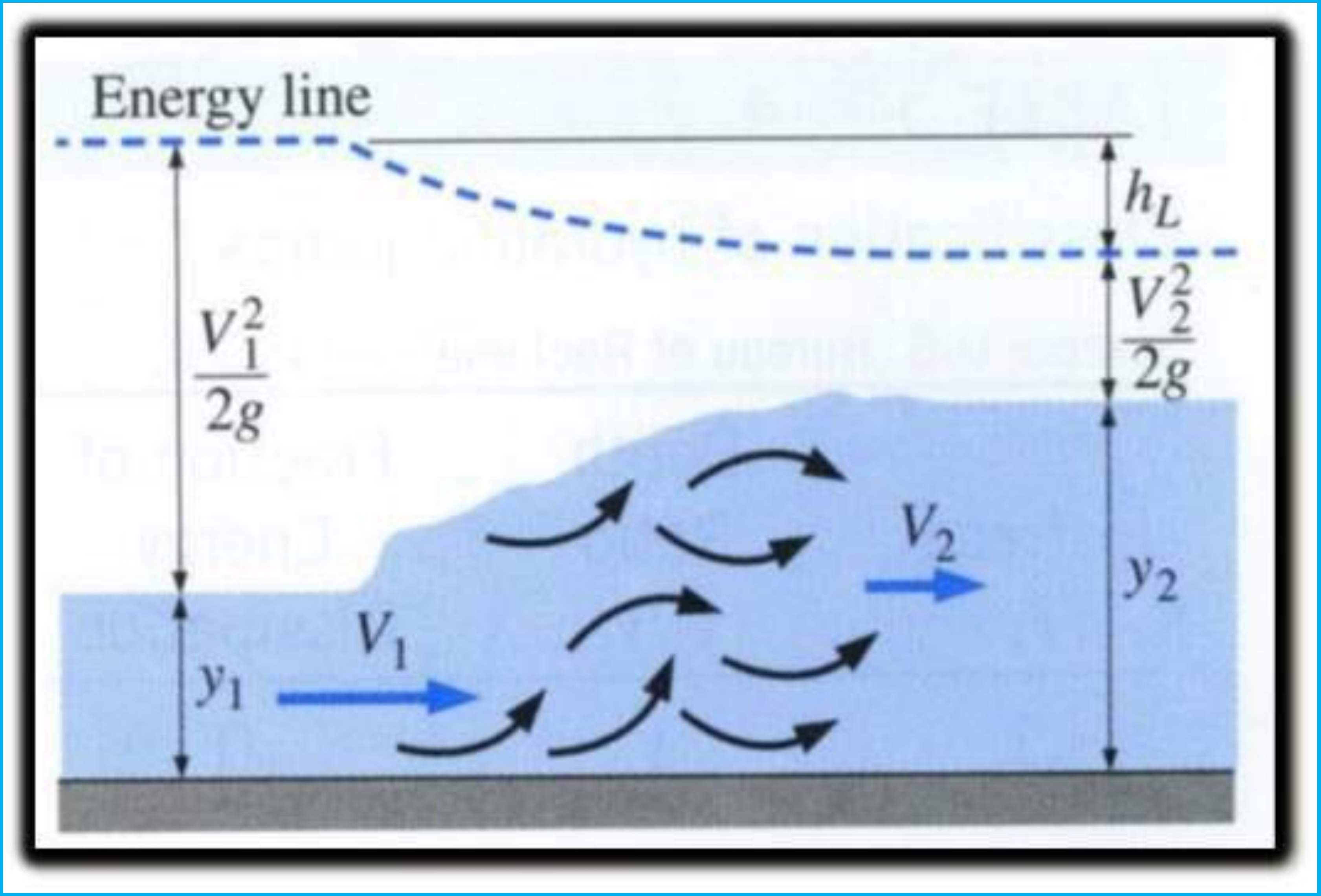
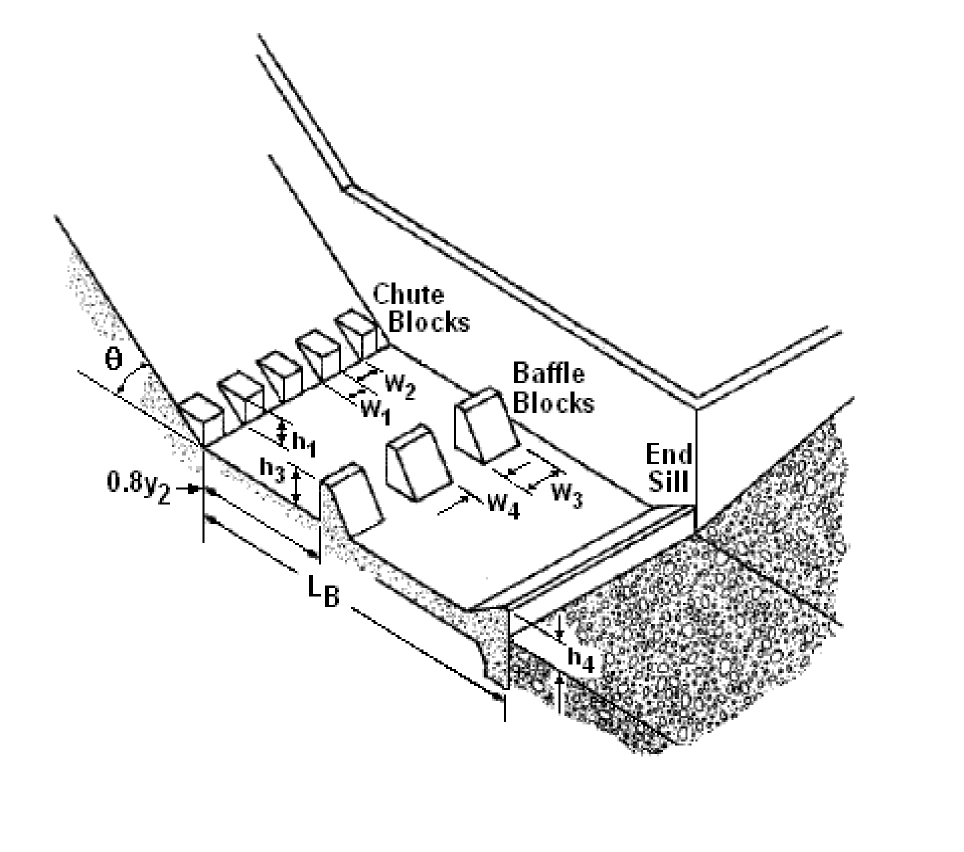
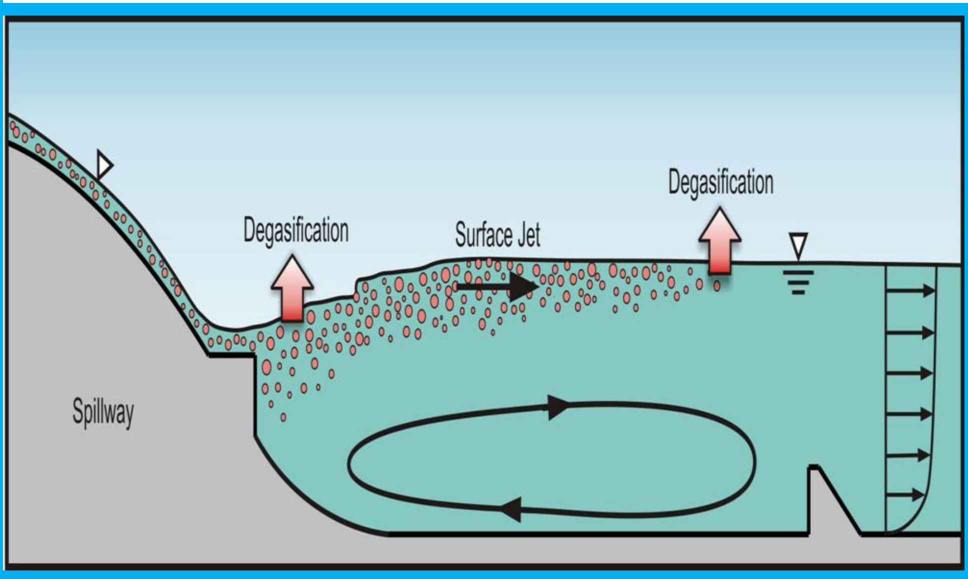


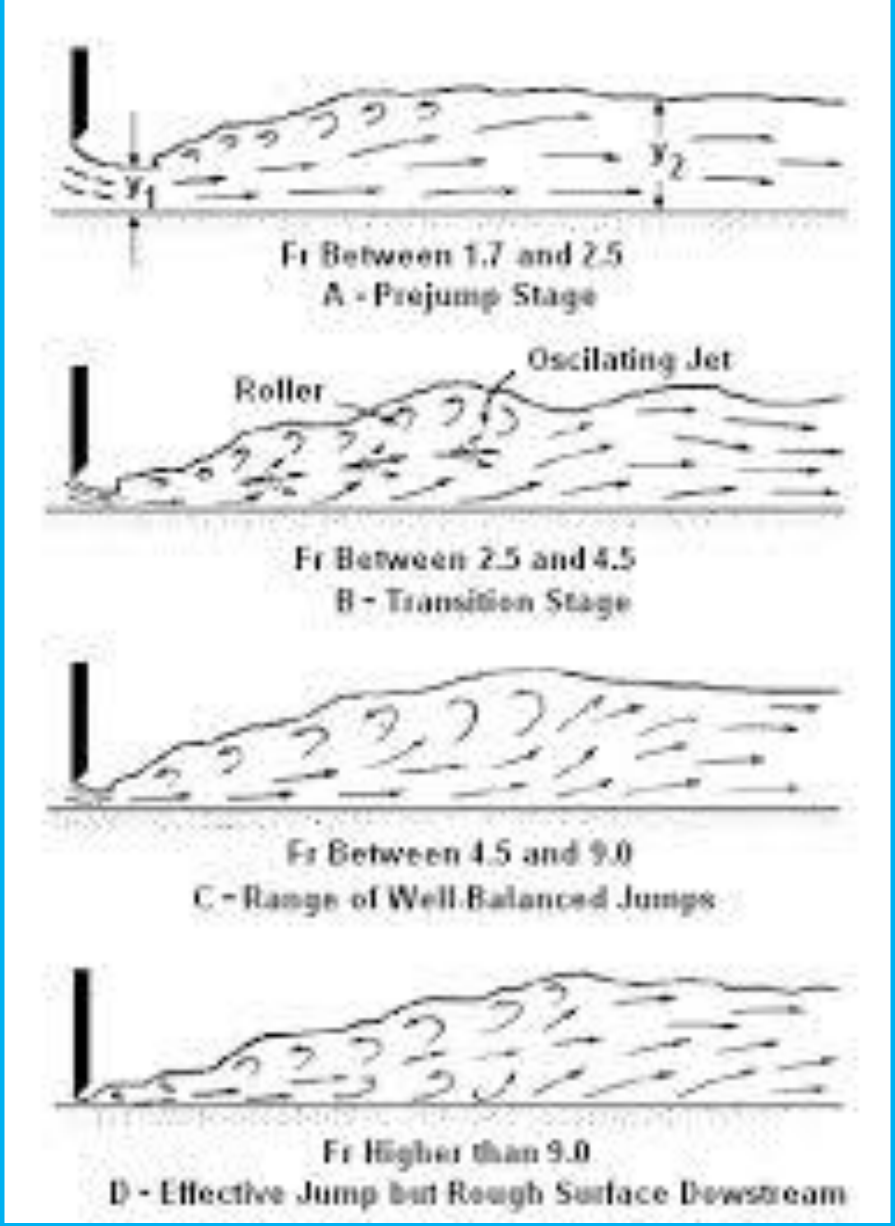
Hydraulic Jump (with Heads and Head Loss)

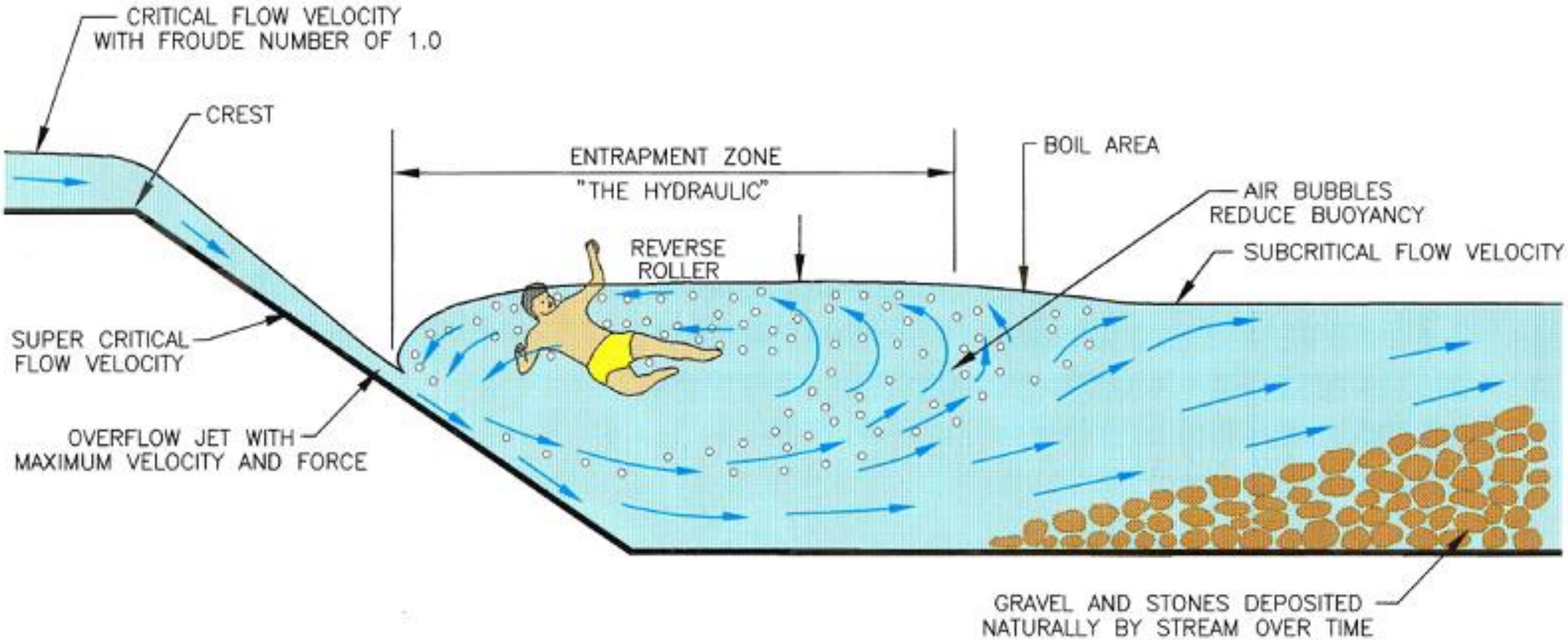
Bernoulli equation in hydraulic jump




Chute
Blocks
Baffle
Blocks
End
Sill
θ
w_2
w_1
h_1
h_3
$0.8y_2$
w_4
w_3
L_B
h_4


Degasification
Surface Jet
Degasification
Spillway


y_1
y_2
Fr Between 1.7 and 2.5
A - Prejump Stage
Roller
Oscilating Jet
Fr Between 2.5 and 4.5
B - Transition Stage
Fr Between 4.5 and 9.0
C - Range of Well Balanced Jumps
Fr Higher than 9.0
D - Effective Jump but Rough Surface Downstream



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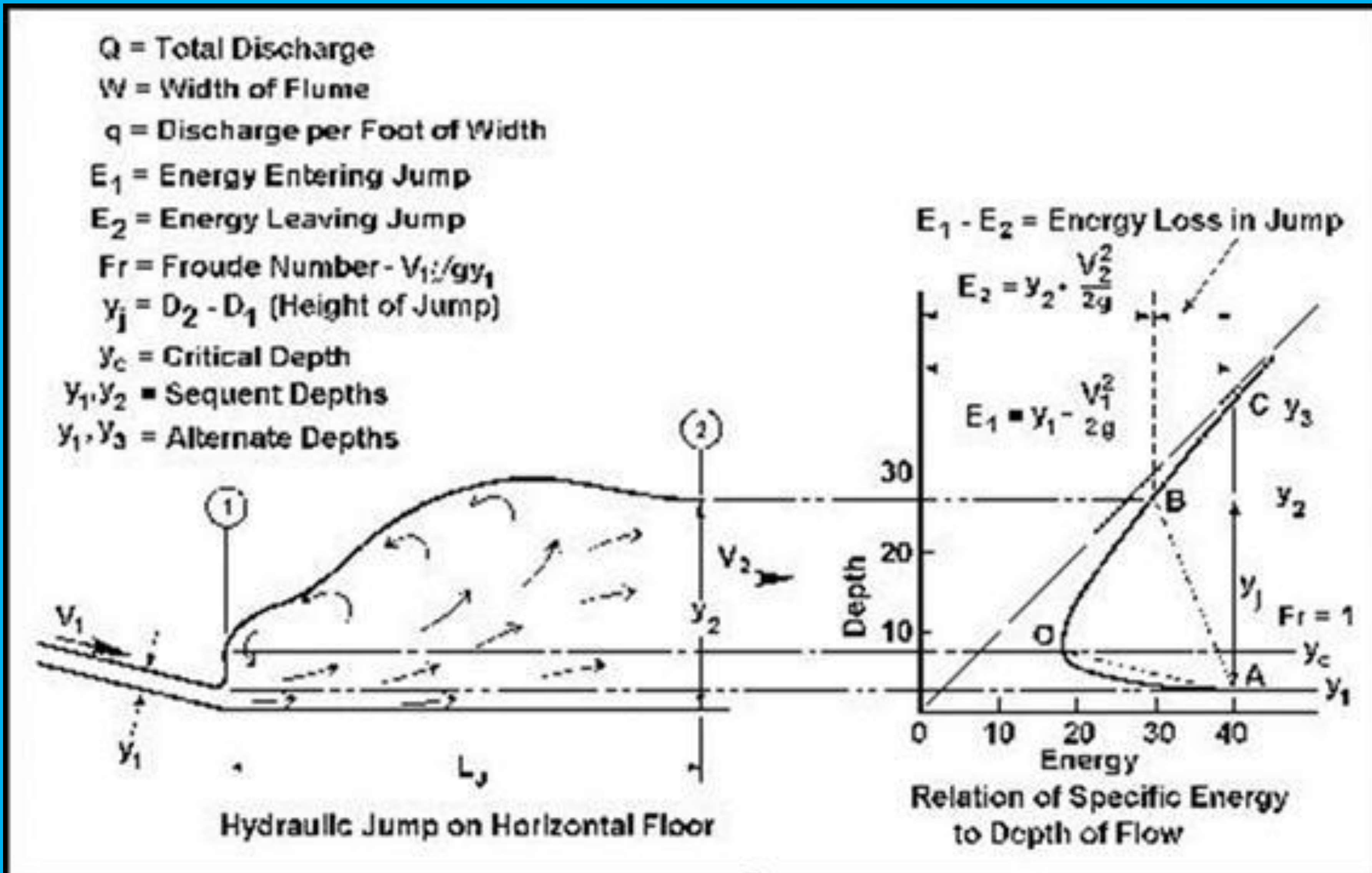
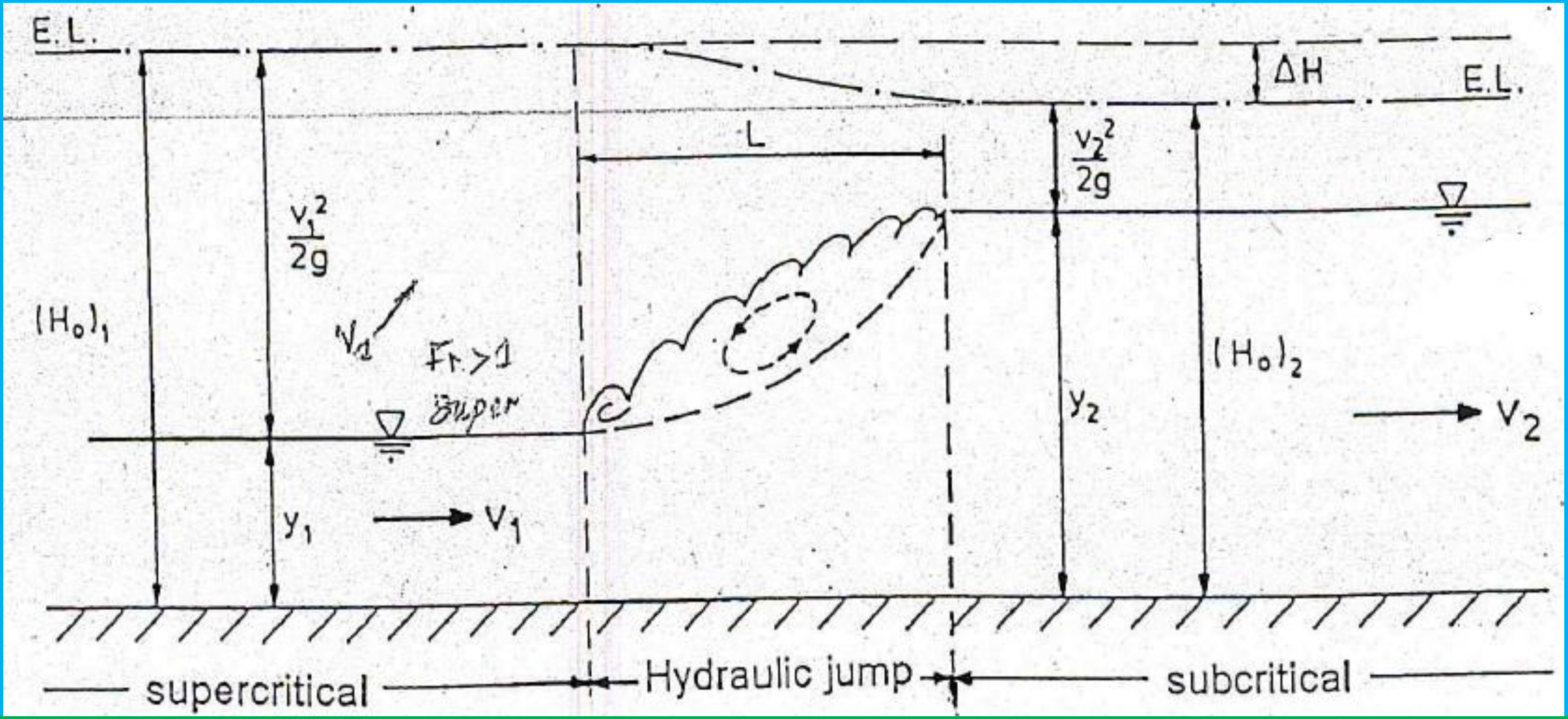

Q = Total Discharge
W = Width of Flume
q = Discharge per Foot of Width
E1 = Energy Entering Jump
E2 = Energy Leaving Jump
Fr = Froude Number - V1/√gy1
yj = D2 - D1 (Height of Jump)
yc = Critical Depth
y1, y2 = Sequent Depths
y1, y3 = Alternate Depths
1
2
V1
y1
V2
y2
Lj
Hydraulic Jump on Horizontal Floor
E1 - E2 = Energy Loss in Jump
E2 = y2 + V2²/2g
E1 = y1 + V1²/2g
Depth
Energy
0
10
20
30
40
O
A
B
C
y3
yj
Fr = 1
yc
Relation of Specific Energy to Depth of Flow

Hydraulic jump:



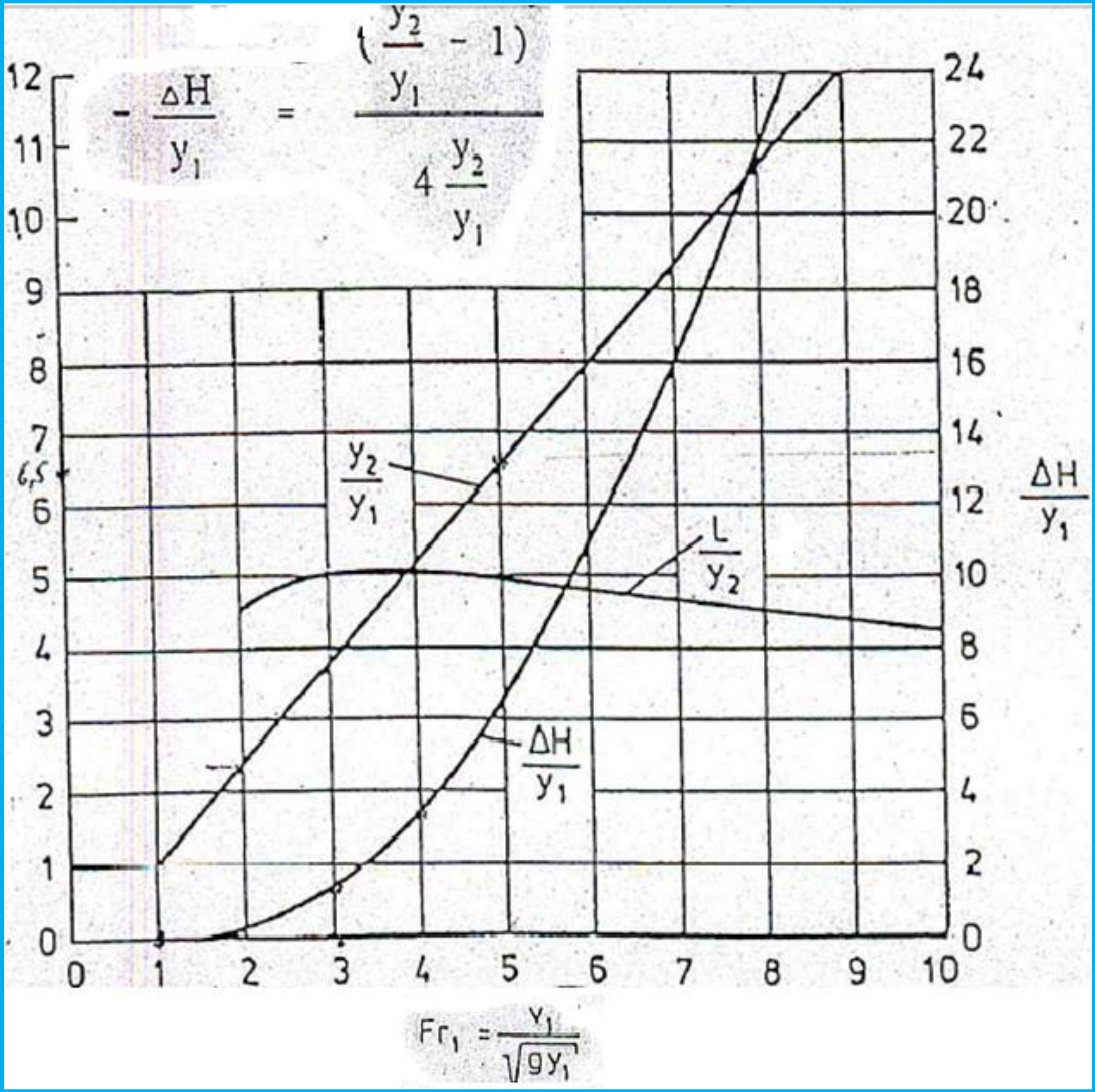
For horizontal rectanglar channel the combination of continuity and momentum equation provides:

$$Fr_1 \equiv \frac{v_1}{\sqrt{g\,y_1}} = \left[\frac{1}{2}\frac{y_2}{y_1}\left(\frac{y_2}{y_1}+1\right)\right]^{1/2}$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2}-1\right)$$

$$\frac{y_1}{y_2} = \frac{1}{2}\left(\sqrt{1+8Fr_2^2}-1\right)$$

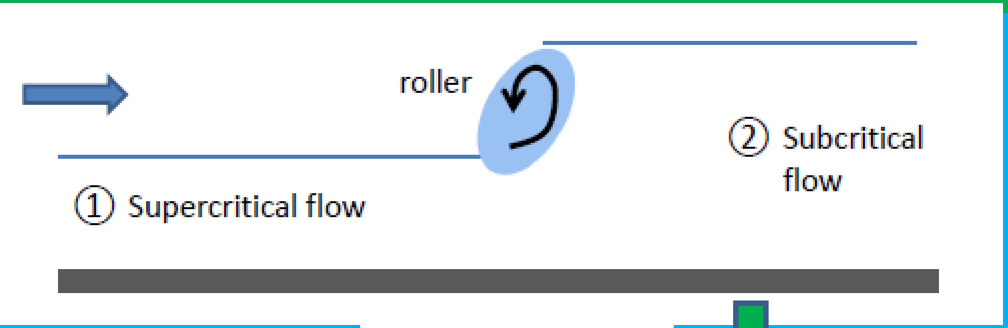
Relation between y_2/y_1 And the Froude number



$-\frac{\Delta H}{y_1} = \frac{(\frac{y_2}{y_1} - 1)^3}{4\frac{y_2}{y_1}}$
$\frac{y_2}{y_1}$
$\frac{L}{y_2}$
$\frac{\Delta H}{y_1}$
$Fr_1 = \frac{V_1}{\sqrt{g y_1}}$

Basic Characteristics of Hydraulic Jump

Roller regime is similar to wave breaker– strong energy dissipation



The upstream Froude number: $\mathrm{Fr}_1 = \dfrac{V_1}{\sqrt{gh_1}} > 1$

For subcritical flow (high flow depth, low flow velocity)

The downstream Froude number: $\mathrm{Fr}_2 = \dfrac{V_2}{\sqrt{gh_2}} < 1$

For supercritical flow (low flow depth, high flow velocity)

Momentum equation

The principle is derived from Newton's second law of motion

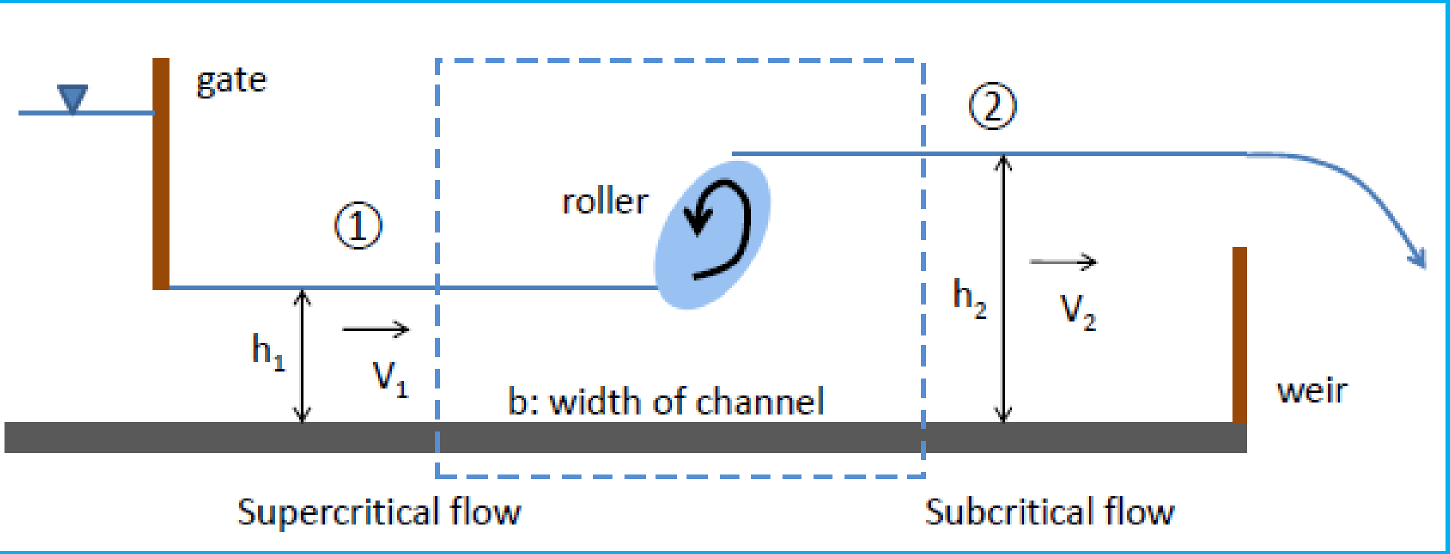
$$F = m.a$$

$$a = \frac{v_2 - v_1}{t}$$

$$Ft = mat$$

$$Ft = m(v_2 - v_1).$$

$$\int_{t_1}^{t_2} F\,dt = m\int_{t_1}^{t_2} a\,dt = m(v_2 - v_1)$$



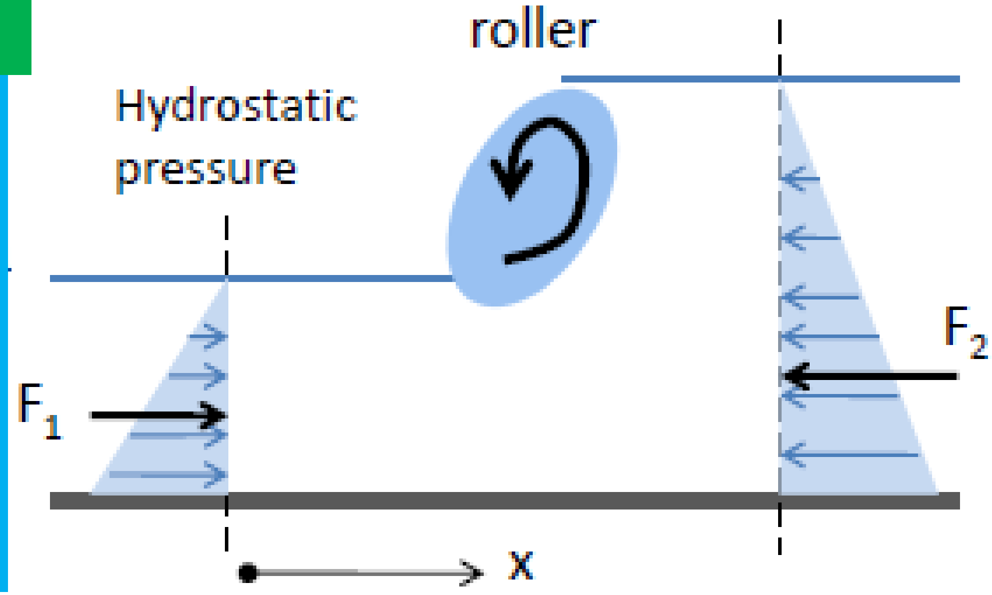
Conservation of mass

$$V_1 h_1 b = V_2 h_2 b \;\; \Rightarrow \;\; V_1 h_1 = V_2 h_2$$

Conservation of Momentum

$$\sum_x F = \sum_{out} \text{Momentum} - \sum_{in} \text{Momentum}$$

$$F_1 - F_2 = \rho Q(V_2 - V_1)$$



If locations (1) and (2) are sufficiently away from the roller, we can assume the pressure distribution is hydrostatic.

$$F_1 = P_c h_1 b = \left(\frac{1}{2}\rho g h_1\right) \cdot h_1 b = \frac{1}{2}\rho g h_1^2 b$$

$$F_2 = \frac{1}{2}\rho g h_2^2 b$$

Recall: Force acting on a submerged surface is equal to pressure at centroid of the submerged area times area of submerged surface.

Momentum conservation

$$\frac{1}{2}\rho g b\left(h_1^2 - h_2^2\right) = \rho V_1 h_1 b\left(V_2 - V_1\right)$$

$$\frac{1}{2} g\left(h_1^2 - h_2^2\right) = V_1 h_1\left(V_2 - V_1\right)$$

Replace *V2 by continuity*

$$\frac{1}{2}(h_1 - h_2)(h_1 + h_2) = \frac{V_1 h_1}{g}\frac{V_1}{h_2}(h_1 - h_2)$$

Recall: by definition:

$$Fr_1^2 = \frac{V_1^2}{g h_1}$$

$$(h_1 + h_2) = 2Fr_1^2 \frac{h_1^2}{h_2}$$

$$\frac{h_2}{h_1^2}(h_1 + h_2) = 2Fr_1^2$$

$$\left(\frac{h_2}{h_1}\right)^2 + \left(\frac{h_2}{h_1}\right) - 2Fr_1^2 = 0$$

set : $Y = \frac{h_2}{h_1}$

$$Y^2 + Y - 2Fr_1^2 = 0$$

We can easily find solution of this 2nd order polynomial
Discard negative root

$$\frac{h_2}{h_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8Fr_1^2}\right)$$

downstream flow depth of a hydraulic jump can be predicted by upstream flow information.

Conservation of energy (to get head loss) Note: *P1=P1=0*

$$h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g} + h_L$$

You can directly get hL1 from measured upstream & downstream flow depth, flow rate

$$h_L = (h_1 - h_2) + \frac{1}{2g}\left(V_1^2 - V_2^2\right)$$

$$\frac{1}{2g}\left(V_1^2 - \frac{V_1^2 h_1^2}{h_2^2}\right) = \frac{V_1^2}{2g}\left(1 - \frac{h_1^2}{h_2^2}\right) = \frac{V_1^2}{2g}\frac{h_1}{h_1}\left(1 - \frac{h_1^2}{h_2^2}\right) = \frac{1}{2}\mathrm{Fr}_1^2 h_1\left(1 - \frac{h_1^2}{h_2^2}\right)$$

$$h_L = (h_1 - h_2) + \frac{1}{2}\mathrm{Fr}_1^2 h_1\left(1 - \frac{h_1^2}{h_2^2}\right)$$

$$\frac{h_L}{h_1} = \left(1 - \frac{h_2}{h_1}\right) + \frac{1}{2}\mathrm{Fr}_1^2\left[1 - \left(\frac{h_1}{h_2}\right)^2\right]$$

Recall **previously**:

$$\mathrm{Fr}_1^2 = \frac{1}{2}\left[\left(\frac{h_2}{h_1}\right)^2 + \left(\frac{h_2}{h_1}\right)\right] \qquad Y = \frac{h_2}{h_1} \qquad \therefore\ \frac{1}{2}\mathrm{Fr}_1^2 = \frac{1}{4}\left(Y^2 + Y\right)$$

Hence,

$$\frac{h_L}{h_1} = (1-Y) + \frac{1}{4}(Y^2+Y)\left(1-\frac{1}{Y^2}\right)$$

$$= \frac{1}{4Y}(Y^3 - 3Y^2 + 3Y - 1)$$

$$= \frac{(Y-1)^3}{4Y}$$

Substitute h2, h1 back

$$h_L = \frac{(h_2 - h_1)^3}{4h_2h_1}$$

hL obtained theoretically can be compared with directly measured value **hL1**

Height of hydraulic jump *(h_j)*:

The difference of depths before and after the jump is known as the height of the jump,

$$h_j = y_2 - y_1$$

Length of hydraulic jump *(L_j)*:

The distance between the front face of the jump to a point on the downstream where the rollers (eddies) terminate and the flow becomes uniform is known as the length of the hydraulic jump. The length of the jump varies from 5 to 7 times its height. An average value is usually taken:

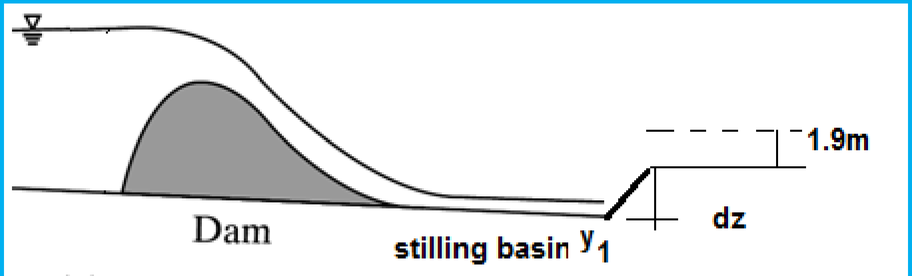
$$L_j \simeq 6 h_j$$

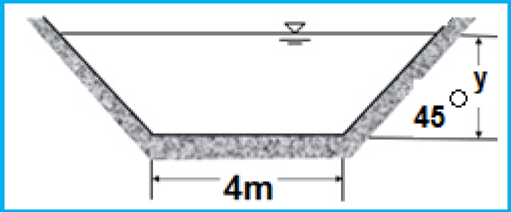
Exercice 7

Design the height dz of the sill in a way that hydraulic jump is located within the stilling basin

Outline the course of the water level and the energy line

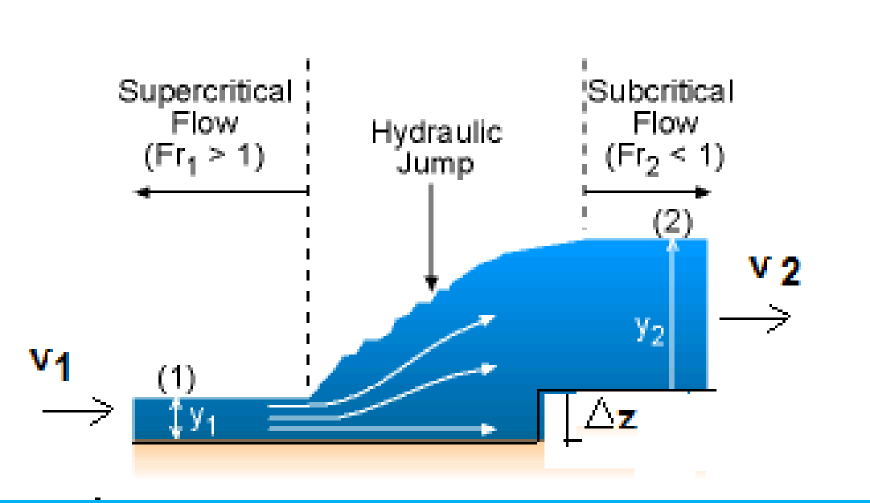
$Q = 36 \frac{m^3}{s}$ $y_1 = 0.53m$

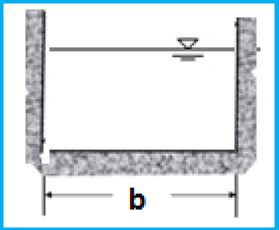




Exercice 8

Design the height dz of the sill of the hydraulic jump
using these values bellow considering a rectangular channel.





$$v_1 = 8\frac{m}{s}$$

$$y_1 = 1m$$

$$y_2 = 2.5m$$