Praise Worthy Prize

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Equivalents Methods to Investigate Free Vibration of Isotropic and Orthotropic Thin Rectangular Plate with Non-Homogeneous Supports

A. Sekhri[1], T. Benmansour[2], H. Cheridi[2]

Abstract – *Evaluating approximate frequency for isotropic and orthotropic plates is a complicated problem, thus exploiting the general formula of Hearmon, it is proposed in this investigation to calculate the fundamental mode of isotropic and orthotropic plates with two non-homogenous supports (cases: SCSC & SSCC). For the higher mode of modal frequency, a particular form of Rayleigh's method is used leading to a simple procedure for calculating the fundamental frequency. A new simple and qualitative method is proposed and has permitted a good strategy to evaluate the quality of results obtained. In order to verify the precision of the proposed qualitative method, a confrontation with finite element method using ANSYS software was done. The complementary utilization of Hearmon's principle and the qualitative method has also permitted a successful advance in evaluating higher modes. This combined procedure gives a strategy of a vibratory analysis of isotropic and orthotropic plates; it permits the satisfaction of the preliminary conception needs of the structure to be studied, and also provides a qualitative method for expertise and investigation of dynamical responses. In all cases, the dynamic investigation is based on some evaluation criteria such as: limit conditions effect, plate dimensions ratio effect, material effect and mode number effect.*

Keywords: *Free Vibration, Rectangular Thin Plates, Isotropic-Orthotropic, Circular Frequency-Nodal Lines Position - Qualitative Analysis Method, Rayleigh Method, Finite Element Method*

Nomenclature

E_1, E_2, E_3	Young's modulus in bending for x, y and z direction respectively
G_{12}	Shear modulus in bending for xy plane
υ_{12}	Poisson's ratio corresponding to compressive strain in x direction due to extensional stress in y direction
υ_{21}	Poisson's ratio corresponding to compressive strain in y direction due to extensional stress in x direction
a	Length of side parallel to x-axis
b	Length of side parallel to y-axis
h	Plate thickness
ω	Circular frequency
f	Frequency equal to $\omega/2\pi$
ρ	Mass density of the material
W	Transverse displacement of a point on the plate along z direction
t	Time
i	Number of half waves in x direction
j	Number of half waves in y direction
M	Bending moment reaction for clamped edges
S	Simply supported support
C	Clamped support
x, y, z	Axis of the reference system

I. Introduction

Structural engineering such as civil, mechanical, aeronautical, aerospace, and naval, sports equipment and military using orthotropic plates is common in all these fields. The frequent use of such structures requires an investigation of orthotropic plates to develop an exact and confident conception. In engineering, the finite element method (FEM) provides a complete solution to the problem of evaluation of vibration modes and dynamic responses of an orthotropic plate where the properties of materials and limit conditions are known. However, during the early stages of development and design of the project, it is very useful to have a simplified method to calculate the modal frequency of orthotropic rectangular plates. The essential task of this project was to select the dimensions and properties of materials, as well as applying control quality for design accuracy by finite element method (FEM) means. The problem of free vibration of orthotropic rectangular plates has been intensively studied during the last six decades by several researchers from different nationalities. It began with the

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article of Hearmon [1] who initiated the study of some particular cases; it is possible to find a large number of contributions to the solution of the problem, using different techniques, which apply to plates with a variety of edge and form conditions. Hence, it is theoretically possible to calculate exact solutions of frequency, only for the case of a plate with simply supported sides. For this reason, considerable efforts are made to develop approximate methods with more accuracy. Leissa and Bert [2]-[8] have summarised series of articles on the dynamic behaviour of composite plates and sandwiches while the use of finite elements model is described in reference [9]. A review of scientific literature in this field shows that, because of its conceptual simplicity [10], one of the most popular methods to get approximate solutions of frequencies for an orthotropic rectangular plate is the method of Rayleigh-Ritz. Hearmon in 1959 [11] proposed an approximate solution for general free vibration of orthotropic plates and applied the method of Rayleigh and used the characteristic functions of beams instead of function shapes of plates, for any combination of conditions clamped or simply supported. His approach is an extended development of Warburton [12] work for all vibration modes of isotropic plates with any boundary conditions combination: free, simply supported, or clamped. The contribution of Marongoni et al. [13] is among the articles in which the method of Rayleigh-Ritz is used. This particularly interesting contribution is a continuation to the method presented by Bazely et al. [14] to calculate lower limits of frequencies for isotopic rectangular plates, in order to evaluate lower and upper limits of natural frequencies of a clamped rectangular plate made from orthotropic materials. They used a combination of Rayleigh-Ritz method for the upper limits and a decomposition method proposed by Bazely for the evaluation of lower limits. This leads to high degree of confident approximate results analysis.

Recently Rossi et al. [15] expanded the analysis to a more complicated case i.e. plates with one or more free edges. The difficulty arises for the case of a free edge boundary condition that prevents the use of the formulation of Rayleigh-Ritz. Rossi et al. have overcome the problem of using an optimized formulation of Rayleigh-Ritz method proposed by Laura et al [16]. The results show an excellent agreement with calculation by FEM, including the case of a concentrated mass.

Gorman [17] applied the superposition method, developed for isotropic plates [18], for the case of clamped plate, which showed a good consistency of the method including the eigen values calculated for a wide range of plates varying in geometries and orthotropic materials types available for use in engineering. The proposed method has attracted the attention of several investigators who searched the differential equation which satisfy exactly all plates and boundary conditions with a satisfying degree of precision. In fact, Li [19] showed that it is desirable also to analyze the forced vibration of orthotropic plates, while Mossou and Nivoit [20] used the method to provide a reliable non-destructive testing method to determine elastic constants of an orthotropic material forming a rectangular plate free of vibration.

In Reference [21], a solution in a double trigonometric series is introduced to solve the forced vibration problem of clamped orthotropic plates using an iterative method for analyzing free vibration. Sakata et al. [22] applied an iterative method deduced from a differential equation obtained for a plate already studied in isotropic cases [23], in order to obtain natural frequencies of orthotropic rectangular plates with accurate results. They confirm that the method is simpler than other methods available in literature such as, for example, Rayleigh-Ritz method that requires a large effort of calculation. In Reference [24], the possibility of using approximate solutions for calculating the frequency of anisotropic plate is investigated. In fact, the authors compared the frequency of an anisotropic plate simply supported, calculated using an exact numerical analysis [25], and according to the exact solution of simply supported orthotropic plates. The problem of clamped edges of plates is studied by comparing the frequencies of anisotropic plates with the values obtained using Huber orthotropic and formulas applied for isotropic plates [26]. It is noticed, according to an engineering point of view, that these two approximations gave enough accurate values. Finally, another approach is used by Chen [27] to calculate the fundamental vibration frequency of an orthotropic plate. He used an iterative approach based on finite difference equations and numerical integration, which showed that with less effort of calculation, it is possible to obtain good precision compared to other numerical technologies.

All methods described above involve a considerable effort of calculation; that is why they are not desirable:

1. To define a rapid approximate method;
2. To execute preliminary project designs in the first phase of development or the last general verification for rapid precision.

For this reason, a simplified method for evaluating natural frequencies of an orthotropic plate is invested in this work. The method is based on a formula developed in a general form, using the results obtained by researchers who have approached the problem in recent decades.

Based on a general expression of fundamental frequency of a plate, suggested by Hearmon [1] and based on three factors that take into account the boundary conditions, it is possible to access higher modal frequencies, by means of a numerical procedure that uses a formulation of Rayleigh method. This reduces the problem of evaluating any frequency of orthotropic plate on the basis of the fundamental frequency of an equivalent plate associated with the real one. A similar approach is used by Sakata [28] to study higher natural frequencies, but it is limited to the case of an isotropic rectangular plate simply supported along both sides, parallel to the y-axis, and retained against restraint rotation along the other side parallel to x-axis.

A. Qualitative method of dynamic analysis of rectangular plates:

Qualitative method is based on theory of long plates (infinite), whose behaviour is dominated by privileged little sense of rectangular plate. In this case, the plate behaves like an equivalent beam, based on its two supports along the long plate.

By simple observation and on basis of fixed conditions as well as the dimensions ratio (L_x/L_y), we can quickly appreciate comparative study of both upper and lower rigidities and consequently the extreme lower and upper frequencies, which limit dynamic answers within tolerable limits and thus judge the quality of results that can be achieved according to a more sophisticated analysis.

B. Approximate quantitative analysis of dynamic rectangular plates:

In this work problem of approximating frequencies for orthotropic plate is investigated. Based on a general formula of approximate frequency, as proposed by Hearmon, the calculation of basic mode is shown for an orthotropic rectangular plate with different fixity conditions, using **coefficients values** that already exist in scientific literature [1]. In addition to higher modes of frequencies, a particular form of Rayleigh method is proposed, leading to a simple procedure for calculation of fundamental frequency. In fact, calculation of higher frequencies is reduced to **the evaluation of fundamental frequency of a specific equivalent plate associated with the actual original one.**

In all cases, dynamic investigation is based on some evaluation criteria including boundary conditions effect; plate dimensions ratio effect, material effects and mode number effect. To verify the accuracy of the proposed method, a confrontation with finite element method ANSYS is done.

II. Problems and Applications

Mathematical models and corresponding solutions of a plate simply supported on its four edges are cited from classical theory [29] and some numerical methods are considered to handle the solution of other cases of limit conditions. Accurate determination of frequencies, outside simply supported case presents difficulties in the integration of differential equation of dynamics motion of 4^{th} order. That is why the use of approximate methods for the purpose of practice calculation of frequency is necessary:

For this reason, in this work, a simplified method to evaluate the natural frequency of an orthotropic plate is investigated. The method is based on an approximate formula developed in a general form, using results obtained by researchers who have approached the problem in recent decades. Based on the general expression of fundamental frequency of plate, as suggested by Hearmon [1] and based on three factors that take into account boundary conditions, it is possible to access higher modal frequencies, by means of a numerical procedure, which uses a specific formulation of Rayleigh method. This reduces the problem of evaluating any frequency of an orthotropic plate based on fundamental frequency of an equivalent plate associated with the real one. An orthotropic material is characterized by the fact that elastic mechanical properties have two symmetrical plans thus only four independent elastic constants specifically E_1, E_2, G_{12}, υ_{12} are considered. Coefficient υ_{21} can be determined using equation:

$$\begin{aligned} &\frac{E_1}{E_2} = \frac{\nu_{12}}{\nu_{21}}, \\ &D_1 = \frac{E_1 h^3}{12\mu}, D_2 = \frac{E_2 h^3}{12\mu}, \\ &D_{12} = \frac{G_{12} h^3}{12}, \mu = 1 - \nu_{12}\nu_{21}, \\ &2H = \nu_{21} D_1 + \nu_{12} D_2 + 4 D_{12} \end{aligned} \tag{1}$$

The hypothesis of Love-Kirchoff which neglects shear force and rotary inertia effect with reference to Fig. 1 [30] is used:

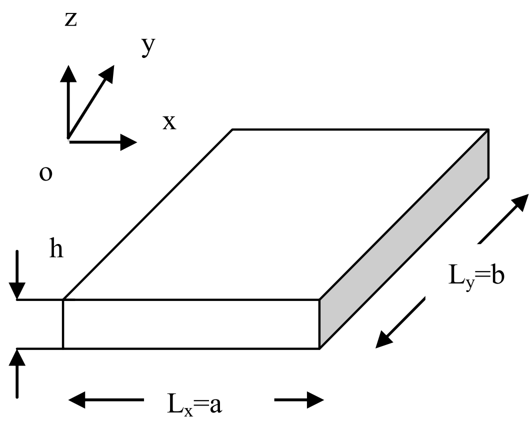


Fig. 1. Model adopted in the paper

TABLE I
MECHANICAL PROPERTIES [5]

Mechanicals Properties	ISO	ORTHO
E_1 (MPa)	1 E+10	1 E +10
E_2 (Mpa)	1 E+10	6.67 E+09
υ_{12}	0.2	0.25
G_{12} (MPa)	4.17 E+09	3.04 E + 09
ρ (kg/m^3)	7800	7800

TABLE II
GEOMETRICAL PROPERTIES

	Model 1	Model 2	Model 3
L_x (m)	4	4	4
L_y (m)	3	3.2	2.667
Thickness(m)	0.01	0.01	0.01

Hence, motion equation is obtained by:

$$D_1 \frac{\partial^4 w}{\partial x^4}(x,y,t) + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2}(x,y,t) + \\ +D_1 \frac{\partial^4 w}{\partial y^4}(x,y,t) + \rho h \frac{\partial^2 w}{\partial t^2}(x,y,t) = 0 \quad (2)$$

A solution of general form is considered, that is:

$$w = W(x,y)(A\cos\omega t + B\sin\omega t) \quad (3)$$

It is possible to obtain from former equation an expression of two variables only:

$$D_1 \frac{\partial^4 W}{\partial x^4}(x,y) + 2H \frac{\partial^4 W}{\partial x^2 \partial y^2}(x,y) + \\ +D_1 \frac{\partial^4 W}{\partial y^4}(x,y) + \Lambda^4 W = 0 \quad (4)$$

with $\Lambda^2 = \omega\sqrt{\rho h}$ equation (4) must be solved to satisfy the following limit conditions:

M=0; R=0 for Free side (F)
M=0; W=0 for simply supported side (SS)
W=0; $\partial W/\partial x$ (or $\partial W/\partial y$) =0 for Clamped side (C)

III. Discussion of Some Results

III.1. Finites Elements Method

The response according to finite element analysis is based on some models of rectangular meshing with (20x20), (40x40), (80x80) and (120x120). The case of 80x80 has been taken as a reference of comparison [31].

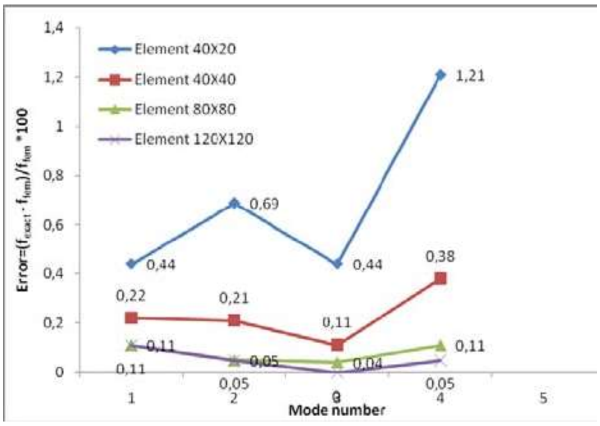


Fig. 2. Evolution of frequency factor with function modal number ($N_1[\omega_{11}]$, $N_2[\omega_{12}]$, $N_3[\omega_{21}]$, $N_4[\omega_{22}]$,$N_5[\omega_{23}]$, $N_6[\omega_{32}]$. Rectangular plate type SSSS, with dimension ratio (a/b) =1.25

The precision of investigation method is based on the study of a plate constitute of one isotropic material and one orthotropic materials. A comparative study shows that the approximate method according to Hearmon is conservative regarding to finite element method. We notice however that the error level is acceptable ($\approx$ - 0.7 %.).

III.2. Dynamic Investigation

Dynamic investigation is based on some evaluation factors such as dimension ratio effect, limit conditions effect, material effect and modal sequence effect. According to these factors, analysis has permitted to apprehend some aspects of plate vibratory behaviour; especially limit conditions over the threshold growing of frequency curves with increasing of supplementary links. It is also noted that the increase of the frequency corrective term related to mode number. The influence of the augmentation of the dimension ratio (L_x/L_y) indicates a parabolic augmentation. Long plates in one direction or another, and regardless of material reduces the contribution of rigidities in long sense which reduces vibration behaviour to that of a beam supported on two elongated boards with fixity conditions of the same edges and the small sense. We have also proposed an original qualitative and quantitative dynamic method, based on the concept of long plates, which are dominated by the behavior of oriented beam according to the small sense of rectangular plate supposed infinite. This qualitative method helps to develop better impression on the assessment of dynamic behaviour of isotropic and orthotropic rectangular plates with non-homogeneous boundary conditions.

According to dimensions ratio of plate (L_x/L_y), the fixity conditions of the equivalent beam (representative of the plate under study), correspond to those on both elongate sides. An approximate solution of vibration analysis of isotropic and orthotropic free rectangular plates is then introduced. Based on a general formulation of approximate frequency proposed by Hearmon, it is shown how to calculate the basic mode of an orthotropic rectangular plate subject to various fixity conditions using coefficient, which is already available in scientific literature.

However, for higher modes, a particular form of Rayleigh method is considered, leading to a simple procedure to calculate fundamental frequencies. In fact, the calculation of frequencies is reduced to the evaluation of the fundamental frequency of a special plate associated with the existing original one.

Discussion: the influence of dimensions ratio (a/b) effect and the fixity conditions combination on the behavior of free vibration of orthotropic rectangular plates.

In the following graphs, we will study the evolution of the natural frequency factor under double influence of dimensions ratio and fixity conditions.

Orthotropic plates subjected to the limit conditions:

- **SCSC case**
- **SSCC case**

Figure 3 shows the growing influence of dimensions ratio on the growing curves for two cases of fixities SCSC and SSCC, on the other hand the influence of boundary conditions can be observed by comparing the upright dispersion of these two curves.

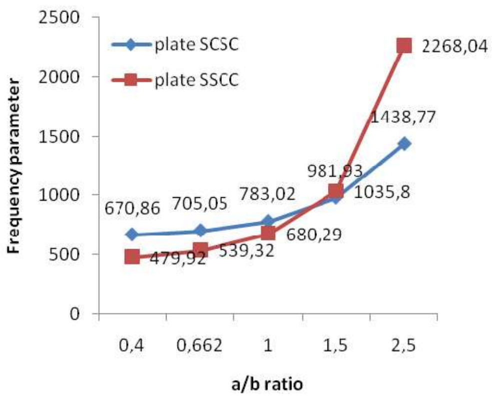


Fig. 3. Effect of dimensions ratio a/b & condition of fixities SCSC and SSCC on factor evaluation of orthotropic rectangular plates frequencies

It is noted that according to forefront of dimensions ratio (a/b), the level of frequency curves was inversed from a certain common ratio [(a/b) ≈ 1.45] which provides a common factor of frequency equal to substantially 944.8 which corresponds to the same type of behaviour despite fixities conditions which are different for these two plate cases. This interesting remark means that there is a limit of a/b for which the two types of rectangular plates SCSC and SSCC remain insensitive to the effect of boundary conditions and behave identically.

There are also two rows for (a/b) ratios, depending on whether we are going in the sense of lower or upper value of the ratio [(a/b) ≈ 1.45].

In fact in one hand frequency curves form an upper limit for the case SCSC and a lower limit for the case SSCC, on the other hand inversed redistribution curves level which gives the opposite effect i.e. respectively to common dimension ratio 1.4, there is a lower limit for SCSC case and an upper limit for SSCC case.

This remark is confirmed by the qualitative analysis (Figure 4) because the plate is evaluated in the direction of beam behaviour dominated by the small sense [(L_x=a) or (L_y=b)] depending on (a/b) ratio.

A comparative analysis of these two plates SCSC and SSCC for a growing evolution of [(a/b) >> 1.45] ratio gives a vibratory behaviour, which is identified with one of two types of equivalent beams respectively with fixity conditions (SCSC→CC) and (SSCC→CS), the qualitative observation proves that the level of frequency submit condition ($\omega_{SSCC}=\omega_{SC} < \omega_{SCSC}=\omega_{CC}$) and confirms the data of the graph.

In the case of [(a/b) << 1.45], the ratio leads to cases with equivalent beams of fixity conditions (SCSC→SS) and (SSCC→SC) respectively ($\omega_{SSCC}=\omega_{SC} > \omega_{SCSC}=\omega_{SS}$), which further confirms the given graph.

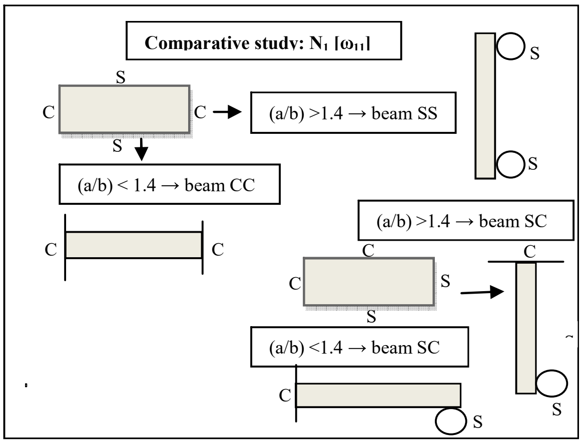


Fig. 4. Qualitative vibratory analysis of rectangular plates SCSC & SSCC according to elongated plate's concept (mod1)

This analysis allows us to see not only the effect of rectangular plate dimension ratio and the effect of support conditions, but also the distribution of the same support conditions.

In fact, two types of fixities are used C and S, but in a different way, which gives the comments previously discussed.

Orthotropic plate subjected to limit conditions of the following types; model 2:

- **Case SCSC**
- **Case SSCC**

This application shows for these two frequency curves, the growing of the corrective term linked to mode number. In one hand, these two plates start from the same value for fundamental mode N_1 (ω_{11}=1.02) which is observed for vibratory qualitative analysis (Figure 5) concerned with the effect of the dimensions ratio [(a/b) = (4/3.2) = 1.25], in the other hand, it is noticed an alternative rollover of frequency factors gradually as evolution growing of modal number N_2, N_3, N_4, N_5. In order to justify this behaviour, qualitative analysis exploiting also approximate method of Hearmon, which is useful for upper modes, is used.

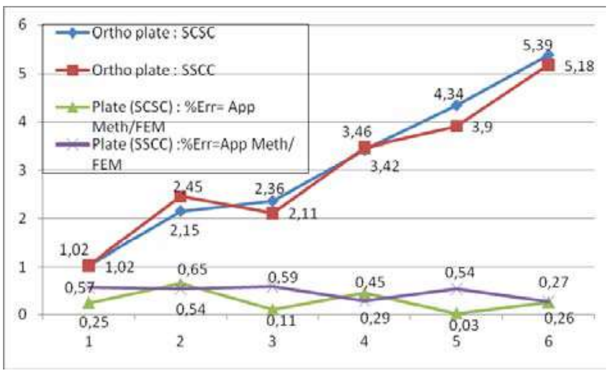


Fig. 5. Evolution of circular frequency according to modal number ($N_1[\omega_{11}]$, $N_2[\omega_{12}]$, $N_3[\omega_{21}]$, $N_4[\omega_{22}]$, $N_5[\omega_{31}]$, $N_6[\omega_{32}]$, and conditions of fixities of plates SCSC & SSCC

Justification of mode N_2 [ω_{12}]:

We should start to count from the bottom side to be consistent with the graph Figure 5, and relating figure 6. Now, if we focus on the sub plate (a/b' = 8/3.2), generated by the mode of the considered sequence N_2 (ω_{12}) (see Figure 6). We can observe, knowing the dimensions ratio of the original plat [(a/b) =4/3.2=1.25] model 2, and based on the theory of vibration of infinite plate, that the behavior should be dominated by the small side (l_y=b). Two types of contributions will be discussed each case apart:

Plat type SCSC

Since the dimension ratio for the two small modal plates (a/b'=8/3.2) generated is still within the conditions to keep the vibration behavior under the control of the small side along the y direction (l_y=b'= b/2), we get vibration contribution of two types of the same beams, both having clamped and roller support [SCSC→2 (SS)]. Figure 6 show how those beam are restrained.

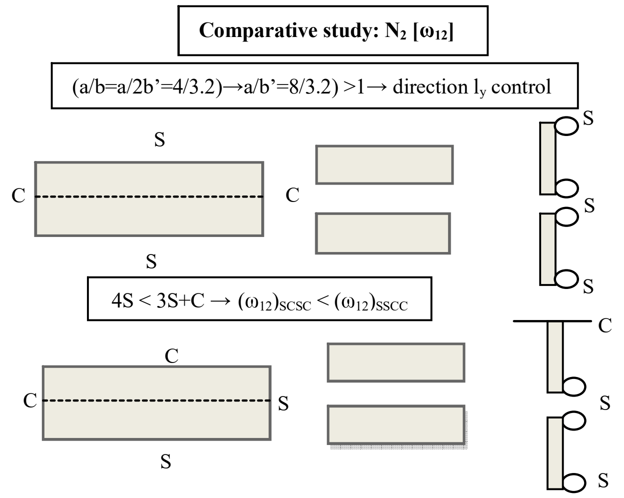


Fig. 6. Qualitative vibratory analysis rectangular plates SCSC- vs- SSCC according to elongated plate's concept (mod2)

We should notice the essential feature of the nodal line, that it does not prevent rotation, however it restraint the beam from translating vertically. By summing the number of restraints involved, we can count:

4 Rollers → (4S)

In the same manner we can show for the following case.

Plat type SSCC

We get vibration contribution of two types of beams, one having clamped and roller support and the second having pin support at one end and a roller support at the other [SSCC→ (CS) + (SS)]. Figure 6 show how those beam are restrained.

1 Clamps + 3 Rollers → (1C+3S)

Finally, let us compare the two cases:

Plat type SCSC:

Mode N_2 (ω_{12}) involve →4 Rollers → (4S)

Plat type SSCC:

Mode N_2 (ω_{12}) involve →1 Clamps + 3 Rollers → (1C+3S)

Based on qualitative analysis we can say, since we have (4S) < (1C+3S), we should have $(\omega_{12})_{SCSC} < (\omega_{12})_{SSCC}$ because there is less restraint in plate SCSC compared to SSCC plate type. This result is consistent with that given by the FEM analysis (see Figure 5). The square black point corresponding to SCSC plate type ω_{12} = 2.15 rad/s come then as expected under the circle red point corresponding to SSCC plate type ω_{12}= 2.45 rad/s.

Justification of mode N_3 [ω_{21}]:

We can see from figure 7, that qualitative analysis for mode N_3 [ω_{21}] for both plate cases SCSC & SSCC, developing for the first, a vibratory sequence dominated by vibration behavior of a double beam simply supported of types CS & SC, however, for the second, a vibratory sequence controlled by a vibratory behavior dominated by the combined contribution of two types of beams CS & SS. Finally let us compare the following two cases:

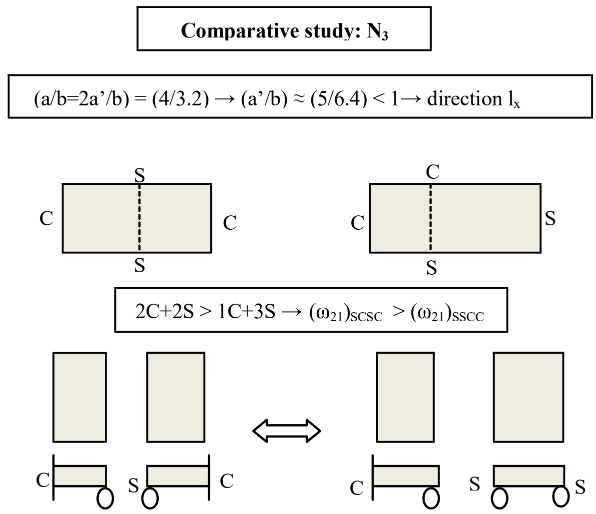


Fig. 7. Qualitative vibratory analysis of rectangular plates SCSC- vs- SSCC according to elongated plate's concept (mod3)

Plat type SCSC:

Mode N_3 (ω_{21}) involve → 2 Clamps + 2 Rollers → (2C+2S)

Plat type SSCC:

Mode N_3 (ω_{21}) involve →1 Clamps + 3 Rollers → (1C+3S)

Based on qualitative analysis we can say, since we have (2C+2S) > (1C+3S), we should have $(\omega_{21})_{SCSC} > (\omega_{21})_{SSCC}$. Rigidity for this sequence modal is higher for plate SCSC compared to that of plate SSCC, because there is more restraint in plate SCSC compared to SSCC plate type. This result is consistent with that given by the

FEM analysis (see Figure 7). The square black point corresponding to SCSC plate type ω_{21}= 2.36 rad/s come then as expected above the circle red point corresponding to SSCC plate type ω_{21}= 2.11 rad/s.

Justification of mode N_4 [ω_{22}]:

Let's compare the two cases:

Plate type SCSC:
Mode N_4 (ω_{22}) involve → 8 Rollers → (8S)

Plat type SSCC:
Mode N_4 (ω_{22}) involve → 6 Rollers + 2 Clamps → (6S+2C)

Based on qualitative analysis we can say, since we have, (8S) < (2C+6S) we should have $(\omega_{22})_{SCSC} < (\omega_{22})_{SSCC}$. We can see the rigidity for this sequence modal are higher for plate SSCC compared to that of plate SCSC, because there is more restraint in plate SSCC compared to SCSC plate type.

This result is consistent with that given by the FEM analysis (see Figure 8). The circle red point corresponding to SSCC plate type ω_{22}= 3.46 rad/s come then as expected above the square black point corresponding to SCSC plate type ω_{22}= 3.42 rad/s.

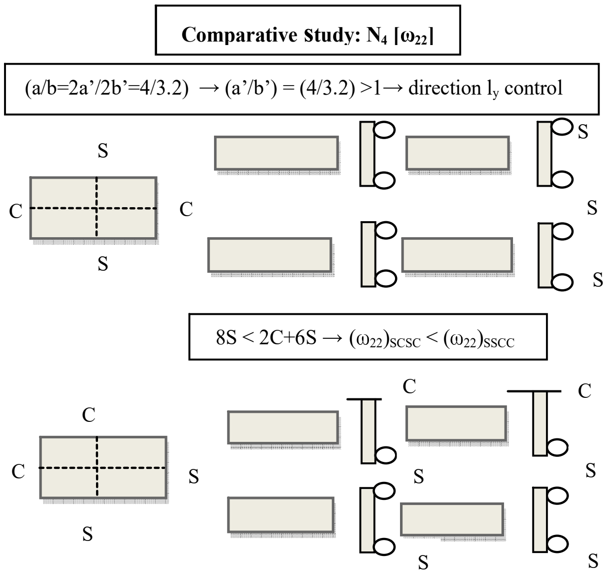


Fig. 8. Qualitative vibratory analysis of rectangular plates SCSC-vs-SSCC according to elongated plate's concept (mod4)

Justification of mode N_5 [ω_{31}]:

Let's compare the two cases:

Plate type SCSC:
Mode N_5 (ω_{31}) involve → 4 Rollers + 2 Clamps → (4S+2C)

Plate type SSCC:
Mode N_5 (ω_{31}) involve → 5 Rollers +1 Clamps → (5S+1C)

Based on qualitative analysis we can say, since we have (2C+4S) > (5S+1C), we should have $(\omega_{31})_{SCSC} > (\omega_{31})_{SSCC}$. We can see the rigidity for this sequence modal are higher for plate SCSC compared to that of plate SSCC, because there is more restraint in plate SCSC compared to SSCC plate type.

This result is consistent with that given by the FEM analysis (see Figure 9).

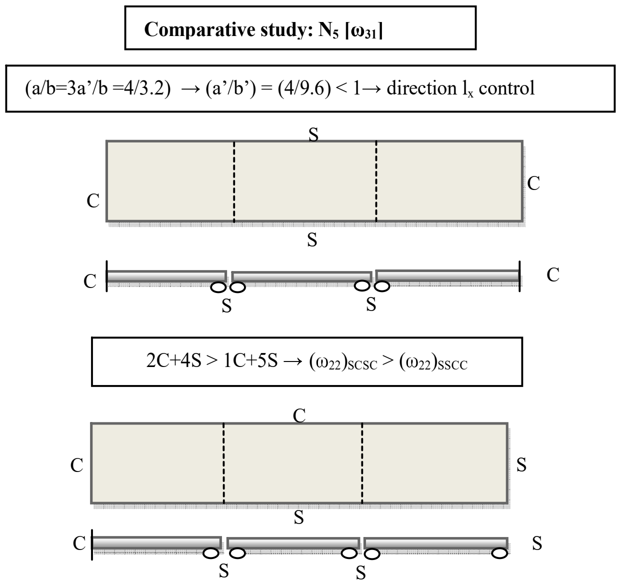


Fig. 9. Qualitative vibratory analysis of rectangular plates SCSC-vs- SSCC according to elongated plate's concept (mod5)

The square black point corresponding to SCSC plate type ω_{31}= 4.34 rad/s come then as expected above the circle red point corresponding to SSCC plate type ω_{31}= 3.9 rad/s.

Justification of mode N_6 [ω_{32}]:

Finally let's compare the two cases:

Plate type SCSC:
Mode N_6 (ω_{32}) involve → 4 Clamps +8 Rollers → (4C+8S)

Plate type SSCC:
Mode N_6 (ω_{32}) involve → 2 Clamps +10 Rollers → (2C+10S)

Based on qualitative analysis we can say, since we have (4C+8S) > (2C+10S), we should have $(\omega_{22})_{SCSC} > (\omega_{22})_{SSCC}$. We can see the rigidity for this sequence modal are higher for plate SCSC compared to that of plate SSCC, because there is more restraint in plate SCSC compared to SSCC plate type.

This result is consistent with that given by the FEM analysis (see Figure 10).

The square black point corresponding to SCSC plate type ω_{32}= 5.39 rad/s come then as expected above the circle red point corresponding to SSCC plate type ω_{32}= 5.18 rad/s.

Note: All these discussion are resumed in Appendix II

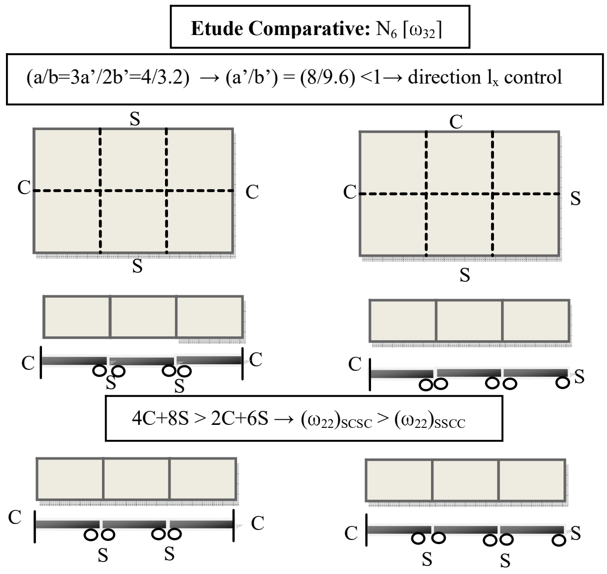


Fig. 10. Qualitative vibratory analysis of rectangular plates SCSC-vs-SSCC according to elongated plate's concept (mod6)

Discussion of percentage of error:

Results obtained with approximate method are confronted to numerical results (FEM) according to the following relation:

$$Error = \frac{(\omega_{FEM} - \omega_{APPR})}{\omega_{FEM}} \cdot 100$$

Errors according to approximate method are overestimated relatively to those obtained by FEM; however, the maximum peak obtained is 0.65%. (See Figure 5 curves green & bleu).

IV. Quantitative Investigation for Higher Frequencies for Orthotropic Plates:

It is important to note for rectangular plates that:

- The nodal line are rectilinear and rectangular to the edges
- They divide the plate in sub plates that vibrate at the same value as ω.
- Nodal line presents zero displacements.

Vibratory analysis of rectangular plate with two cases of fixities (SCSC and SSCC):

The first frequency for orthotropic rectangular plates Hearmon proposed the following equation:

$$f = \frac{\lambda}{2\pi}\sqrt{\frac{D}{\rho h}} \tag{5}$$

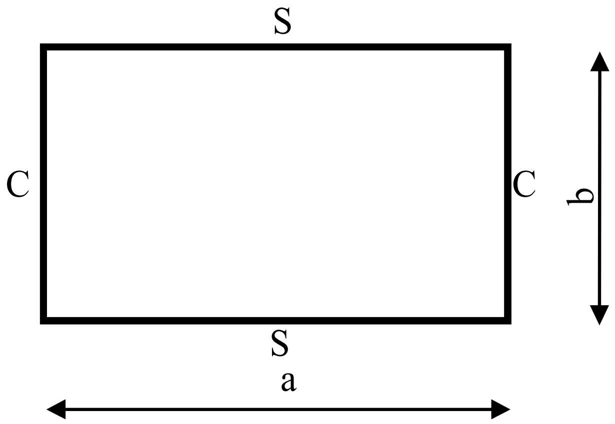
where:

$$\lambda = \sqrt{\frac{A}{a^4} + \frac{B}{b^4} + \frac{C}{a^2 b^2}} \tag{6}$$

Application of the quantitative method based on A, B, C coefficients, which are obtained from a table for the values of λ parameter (see Appendix I).

a) SCSC orthotropic plate:

Mod 1*1



The circular frequency for mod 1*1 is obtained by the following equation:

$$\omega_{1*1} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(4.73)^2 \sqrt{\frac{D_1}{a^4} + \frac{0.195 D_2}{b^4} + \frac{0.485 H}{a^2 b^2}}\right] \tag{7}$$

Mod 1*2

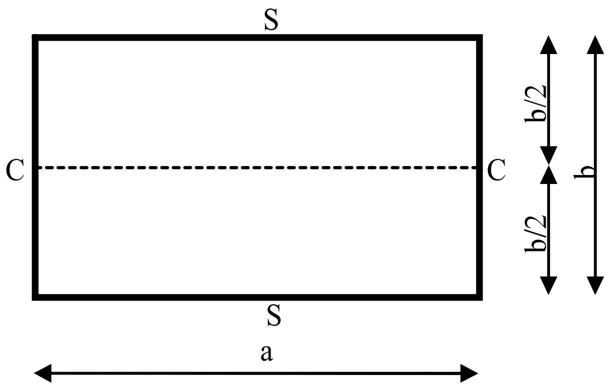


Plate develops a modal sequence with two sub plates SCSC and SCSC where the nodal line position is symmetric because of the symmetry of the extreme supports SS.

The circular frequency for mod 1*2 is obtained by the following equation:

$$\omega_{1*2} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(4.73)^2 \sqrt{\frac{D_1}{(a)^4} + \frac{0.195 D_2}{\left(\frac{b}{2}\right)^4} + \frac{0.485 H}{(a)^2 \left(\frac{b}{2}\right)^2}}\right] \tag{8}$$

Mod 2*1

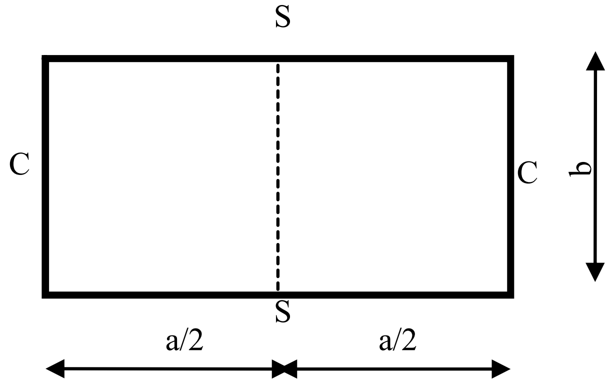


Plate develops a modal sequence with two sub plates SSSC and SCSS where the nodal line position is symmetric because of the symmetry of the extreme supports CC.

The circular frequency for mod 2*1 is obtained by the following equation:

$$\omega_{2*1} = \\ = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(\pi)^2 \sqrt{\frac{D_1}{\left(\frac{a}{2}\right)^4} + \frac{0.195 D_2}{b^4} + \frac{0.485 H}{\left(\frac{a}{2}\right)^2 b^2}}\right] \quad (9)$$

Mod 2*2

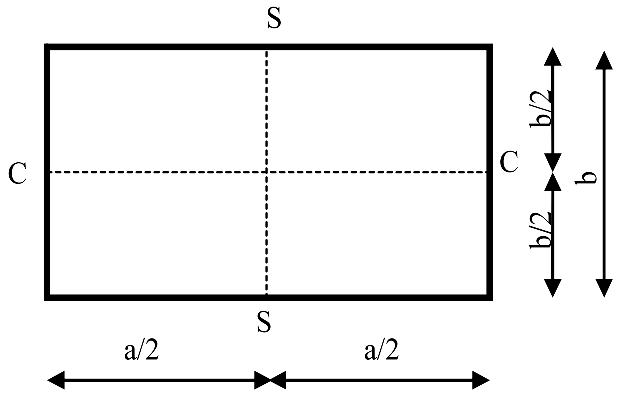


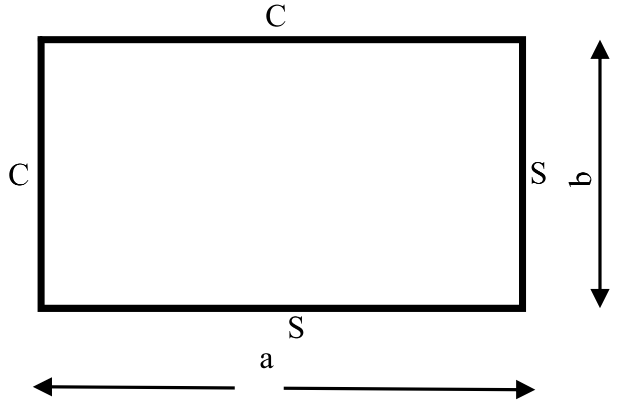
Plate develops a modal sequence with four sub plates 2 (SSSC & SCSS) where the nodal lines positions are symmetric because of the symmetry of the extreme supports CC and SS.

The circular frequency for mod 2*2 is obtained by the following equation:

$$\omega_{2*2} = \\ = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(\pi)^2 \sqrt{\frac{D_1}{\left(\frac{a}{2}\right)^4} + \frac{2.441 D_2}{\left(\frac{b}{2}\right)^4} + \frac{2.333 H}{\left(\frac{a}{2}\right)^2\left(\frac{b}{2}\right)^2}}\right] \quad (10)$$

b) SSCC orthotropic plate

Mod 1*1



The circular frequency for the first mod 1*1 is obtained by the following equation:

$$\omega_{1*1} = \\ = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(3.927)^2 \sqrt{\frac{D_1}{a^4} + \frac{D_2}{b^4} + \frac{1.115 H}{a^2 b^2}}\right] \quad (11)$$

Mod 1*2

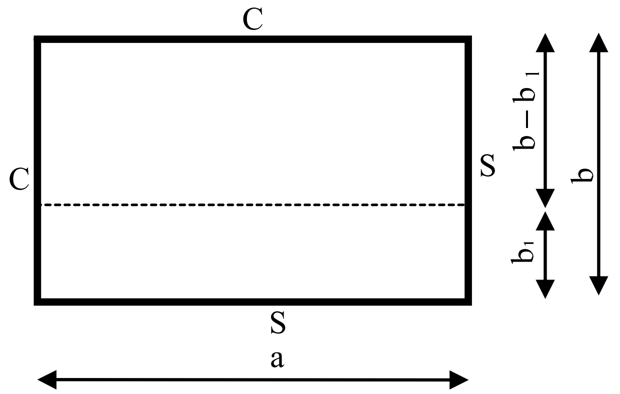


Plate develops a modal sequence with two sub plates SSSC & SSCC where the nodal lines positions are asymmetric because of the no symmetry of the extreme supports CS & CS. The circular frequency for mode 1*2 is obtained by the following two equations:

Sub plate SSCC

$$\omega_{1*2} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(3.927)^2 \sqrt{\frac{D_1}{a^4} + \frac{D_2}{b_1^4} + \frac{1.115 H}{a^2 b_1^2}}\right] \quad (12)$$

Sub plate SSSC

$$\omega_{1*2} = \\ = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(\pi)^2 \sqrt{\frac{D_1}{(a)^4} + \frac{2.441 D_2}{(b-b_1)^4} + \frac{2.333 H}{(a)^2 (b-b_1)^2}}\right] \quad (13)$$

The value b_1 is calculated if we suggest that the two sub plates vibrate at the same frequency $\omega_{1*2(SSCC)} = \omega_{1*2(SSSC)}$.

Mod 2*1

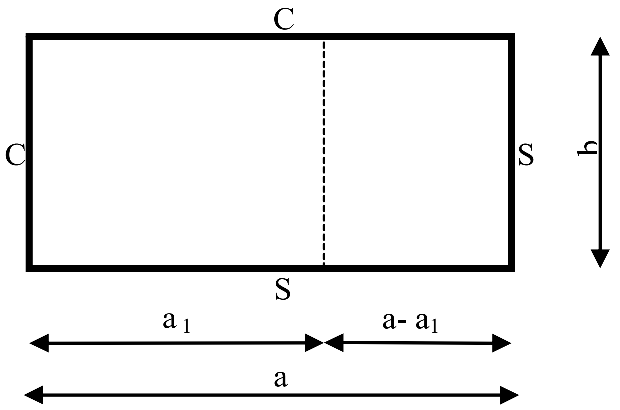


Plate develops a modal sequence with two sub plates SSCC & SSCS where the nodal lines positions are asymmetric because of the non symmetry of the extreme supports CS & CS. The circular frequency for mod 2*1 is obtained by the following equations:

Sub plate SSCC

$$\omega_{2*1} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(3.927)^2 \sqrt{\frac{D_1}{(a_1)^4} + \frac{D_2}{b^4} + \frac{1.115H}{(a_1)^2 b^2}}\right] \quad (14)$$

Sub plate SSCC

$$\omega_{2*1} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(\pi)^2 \sqrt{\frac{D_1}{(a-a_1)^4} + \frac{2.441D_2}{b^4} + \frac{2.333H}{(a-a_1)^2 b^2}}\right] \quad (15)$$

The value a_1 is calculated if we suggest that the two sub plates vibrate at the same frequency $\omega_{2*1(SSCC)} = \omega_{2*1(SSCS)}$.

Mod 2*2

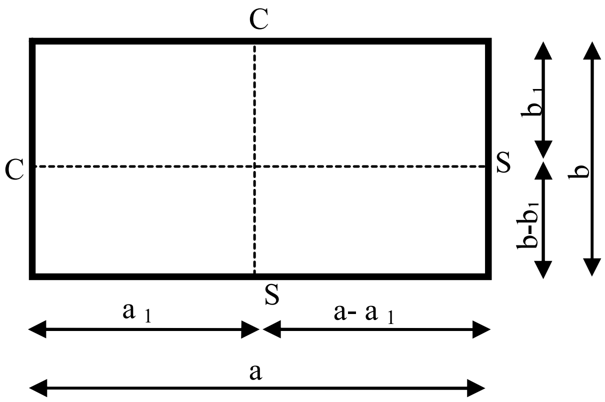


Plate develops a modal sequence with 4 sub plates SSSC, SSCC, SSSS, and SSCS where the nodal lines positions are asymmetric because of the non symmetry of the extreme supports CS & CS.

The circular frequency for mod 2*2 is obtained by the following three equations:

Sub plate SSCC

$$\omega_{2*2} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(3.927)^2 \sqrt{\frac{D_1}{a_1^4} + \frac{D_2}{b_1^4} + \frac{1.115H}{a_1^2 b_1^2}}\right] \quad (16)$$

Sub plate SSCS

$$\omega_{2*2} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(\pi)^2 \sqrt{\frac{D_1}{(a-a_1)^4} + \frac{2.441D_2}{(b_1)^4} + \frac{2.333H}{(a-a_1)^2 (b_1)^2}}\right] \quad (17)$$

Sub plate SSSS

$$\omega_{2*2} = \left(\sqrt{\frac{1}{\rho h}}\right)\left[(\pi)^2 \sqrt{\frac{D_1}{(a-a_1)^4} + \frac{D_2}{(b-b_1)^4} + \frac{2H}{(a-a_1)^2 (b-b_1)^2}}\right] \quad (18)$$

The value a_1 and b_1 are calculated if we suggest that the 4 sub plates vibrate at the same frequency $\omega_{2*2(SSCC)} = \omega_{2*2(SSCS)} = \omega_{2*2(SSSS)} = \omega_{2*2(SSSC)}$.

Note: Quantitative analysis can be extended also for all remaining higher modes.

V. Conclusion and Recommendations

Qualitative method has permitted to support the results quality obtained for different frequency analysis graphs. Complementary use of Hearmon method principal and qualitative method has also permitted to go away with success in evaluating upper modes.

This combined procedure constitutes a strategy of vibratory analysis of isotropic and orthotropic plates; it permits in one hand to satisfy needs of preliminary conception of structure to be studied, but also an investigation and expertise method of dynamic responses quality.

The process of structural analysis can involve risks, especially when using computer.

The paper describes a methodology for handling analysis which also can help to reduce such risks and which promotes an understanding of structural behavior.

Futures recommendations; forced and free analysis may be extended to others cases of fixities and may be a topic of interesting study.

Appendix I

Conditions of fixities	parameter λ (Hearmon, R.F.S)
SSSS	$\lambda = \pi^2 \sqrt{\left(\frac{1}{a^4} + \frac{1}{b^4} + \frac{2}{a^2 b^2}\right)}$
CCCC	$\lambda = (4.730)^2 \sqrt{\left(\frac{1}{a^4} + \frac{1}{b^4} + \frac{0.605}{a^2 b^2}\right)}$
CSCC	$\lambda = (4.730)^2 \sqrt{\left(\frac{0.475}{a^4} + \frac{1}{b^4} + \frac{0.566}{a^2 b^2}\right)}$
SSCC	$\lambda = (3.927)^2 \sqrt{\left(\frac{1}{a^4} + \frac{1}{b^4} + \frac{1.115}{a^2 b^2}\right)}$
SCSC	$\lambda = (4.730)^2 \sqrt{\left(\frac{1}{a^4} + \frac{0.195}{b^4} + \frac{0.485}{a^2 b^2}\right)}$
SSCS	$\lambda = \pi^2 \sqrt{\left(\frac{1}{a^4} + \frac{2.441}{b^4} + \frac{2.333}{a^2 b^2}\right)}$

Appendix II

Mode Number $N_K(\omega_{ij})$	Plate Type	Beam Type Involved By the Sequence		Number of Restraints		Total Number of Restraints Involved	Frequency Ratio (From Qualitative Analysis) $(\omega_{ij})_{SCSC}/(\omega_{ij})_{SSCC}$
N_2 (1-2)	SCSC	SS		2 Rollers (S+S)		4S	2.15/2.45 < 1
		SS		2 Rollers (S+S)			
	SSCC	SS		2 Rollers (S+S)		1C+3S	
		CS		1 Clamp + 1 Roller (C+S)			
N_3 (2-1)	SCSC	CS		1 Clamp + 1 Roller (C+S))		2C+2S	2.36/2.11 > 1
		CS		1 Clamp +1 Roller (C+S)			
	SSCC	CS		1 Clamp + 1 Roller (C+S)		3S+C	
		CS		1 Roller + 1 Roller (S+S)			
N_4 (2-2)	SCSC	SS	SS	2 Rollers (S+S)	2 Rollers (S+S)	8S	3.42/3.46 < 1
		SS	SS	2 Rollers (S+S)	2 Rollers (S+S)		
	SSCC	CS	CS	1 Clamp +1 Roller (C+S)	1 Clamp + 1 Roller (C+S)	2C+6S	
		CS	CS	2 Rollers (S+S)	2 Rollers (S+S)		
N_5 (3-1)	SCSC	CS		1 Clamp + 1 Roller (C+S)		2C+4S	4.34/3.39 > 1
		SS		1 Clamp + 1 Roller (C+S)			
		SC		2 Rollers (S+S)			
	SSCC	CS		1 Clamp + 1 Roller (C+S)		1C+5S	
		SS		2 Rollers (S+S)			
		SS		2 Rollers (S+S)			
N_6 (3-2)	SCSC	CS	CS	1 Clamp+ 1 Roller (C+S)	1 Clamp + 1 Roller (C+S)	4C + 8S	5.39/5.18 > 1
		SS	SS	2 Rollers (S+S)	2 Rollers (S+S)		
		SC	SC	1 Clamp +1 Roller (C+S)	1 Clamp + 1 Roller (C+S)		
	SSCC	CS	CS	1 Clamp+1 Roller (C+S)	1 Clamp + 1 Roller (C+S)	2C + 10S	
		SS	SS	2 Rollers (S+S)	2 Rollers (S+S)		
		SS	SS	2 Rollers (S+S)	2 Rollers (S+S)		

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Author's information

[1]Institutes of Hygiene & Industrial Security,
Batna University, 05000 Batna, Algeria.

[2]Department of mechanical engineering,
Mentouri University, 25000 Constantine, Algeria.

[1, 2]Laboratory of Mechanical construction,
Department of mechanical engineering, Constantine University, Algeria.

A. Sekhri: Constantine, Algeria, march 28, 1961. PhD Student, Civil Engineer 1981, M.phil in Civil engineering 1990, Nottingham University, Great Britain (U.K).
Mechanics of structures, Vibrations of structures, Mechanics of materials, Numerical Methods, Finite element methods, Simulation and numerical modelling, composite materials. Mr. Sekhri, Assistant Professor, at the Institute of Hygiene & Industrial Security, Batna University, 05000 Batna, Algeria.
E-mail: abdelhaksekhri@yahoo.fr

T. Benmansour: Constantine, Algeria, March 6, 1957. Ph.D, Civil engineer 1979, Msc Structural Engineering. *Dept. of Civil Engineering, Oregon State University, Corvallis, Or., USA*, Doctorat d'état in mechanical engineering, Constantine university, Algeria.
Maître de Conférence, Department of mechanical engineering, Mentouri University, 25000 Constantine, Algeria. Mechanics of structures, Vibrations of structures linear and non-lineaires, Mechanics of materials, Numerical Methods, Finite element methods, Simulation and numerical modelling, composite materials
Dr. Benmansour, Maitre de Conference at the Dept of Mechanical Engineering, Mentouri University, 25000 Constantine, Algeria.
E-mail: Toufik.benmansour@yahoo.fr

H. Cheridi, Constantine, Algeria March 09, 1974, Mechanical engineer 2005, Magister 2007 in mechanical engineering, Constantine university, Algeria. Mechanics of structures, Vibrations of structures, Mechanics of materials, Numerical Methods, Finite element methods, Simulation and numerical modelling, composite materials.
Mme Cheridi Ph.D student at the department of mechanical engineering, Mentouri University, Constantine, Algeria.