University of Batna 2
Mathematics and Computer Science Faculty
Common core in Mathematics and computer science

1^{st} year LMD
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Homework * Grps : 15 and 16 *
Algebra 2

Exercise 1

For any $(x_1, x_2, x_3), (y_1, y_2, y_3) \in \mathbb{R}^3$, define an operation of addition by

$$(x_1, x_2, x_3) + (y_1, y_2, y_3) = (x_1 + y_1, x_2 + y_2, x_3 + y_3).$$

and an operation of scalar multiplication by

$$\alpha.(x_1, x_2, x_3) = (\alpha x_1, \alpha x_2, \alpha x_3).$$

Show that $(\mathbb{R}^3, +, .)$ is a voctor space on the field $\mathbb{R}$?

Exercise 2

Given the following vectors of $\mathbb{R}^3$:

$$u = (1, 1, 3), \quad v = (2, -5, 1), \quad w = (0, 2, 3).$$

Compute

1. $(u + v) + w$.
2. $2u - 3v$.
3. $u - v + 3w$.
4. $v + w - u$.

Exercise 3

Determine whether the following sets are subspaces of $\mathbb{R}^3$ under the operation of addition and scalar multiplication defined on $\mathbb{R}^3$. Justify your answers

$$F_1 = \left\{(x, y, z) \in \mathbb{R}^3 / \, 2x + 3y - z = 0\right\}.$$
$$F_2 = \left\{(x, y, z) \in \mathbb{R}^3 / \, 2x^2 = y^2\right\}.$$
$$F_3 = \left\{(x, y, z) \in \mathbb{R}^3 / \, x + 3y - 5y = 2\right\}.$$
$$F_4 = \left\{(x, y, z) \in \mathbb{R}^3 / \, 2x = y, \quad z = -y\right\}.$$
$$F_5 = \left\{(x, y, 0) \in \mathbb{R}^3 / \, xy \leq 0\right\}.$$

Exercise 4

Show that the following sets are bases of the space $\mathbb{R}^3$

1) $\{(-1, 1, 1), (1, -1, 1), (1, 1, -1)\}$.
2) $\{(3, 0, 2), (0, 3, 3), (-1, 0, 2)\}$.

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Exercise 5

Find all values of a such that the vectors u, v and w are independent in $\mathbb{R}^3$, in the following cases :

$$a)\, u = (1, -1, 0),\ v = (a, 1, 0),\ w = (0, 2, 3).$$
$$b)\, u = (2, a, 1),\ v = (1, 0, 1),\ w = (0, 1, 3).$$

Exercise 6

Check if the set $\{u_1 = (2, 3, 0),\ u_2 = (0, 4, 2),\ u_3 = (2, 0, 1)\}$ is a spanning set of $\mathbb{R}^3$. That is, check if $span\,\{u_1,\ u_2,\ u_3\} = \mathbb{R}^3$.

Exercise 7

1. Show that the set $\{u_1 = (0, 1, 4),\ u_2 = (2, 1, 0),\ u_3 = (0, 1, 1)\}$ is a basis of $\mathbb{R}^3$.
2. Find the coordinates of the vector $v = (1, 2, -2)$ with respect to the basis $\{u_1,\ u_2,\ u_3\}$.

Exercise 8

Let F_1 and F_2 be two subsets of $\mathbb{R}^3$ defined by :

$$F_1 = \Big\{(x, y, z) \in \mathbb{R}^3 : \quad x + 2y - z = 0\Big\}.$$
$$F_2 = \Big\{(x, y, z) \in \mathbb{R}^3 : \quad x - y + z = 0, \quad \text{and} \quad 2x - y - z = 0\Big\}.$$

1. Show that F_1 and F_2 are two vector subspaces of $\mathbb{R}^3$.
2. Find a basis and the dimension of the vector subspaces F_1 and F_2.
3. Is $\mathbb{R}^3 = F_1 \oplus F_2$.

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