University year : 2023-2024
Department :MI
Module : Algebra 2

Tutorial Series(1)

Exercise 1 *For any* $(x_1, x_2), (y_1, y_2) \in \mathbb{R}^2$, *define an operation of addition by*

$$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$$

and an operation of scalar multiplication by

$$\forall \alpha \in \mathbb{R},\ \forall (x_1, x_2) \in \mathbb{R}\ ;\ \alpha \cdot (x_1, x_2) = (\alpha \cdot x_1, \alpha \cdot x_2)$$

Is $(\mathbb{R}^2, +, .)$ *a vector space on the field* $\mathbb{R}$ *?*

Exercise 2 *Determine which of the following subsets are subspaces of* $\mathbb{R}^3$. *Give reasons for your answers.*

1. $F_1 = \{(x, y, z) \in \mathbb{R}^3 \mid x - y + z = 0\}$.
2. $F_2 = \{(x, y, z) \in \mathbb{R}^3 \mid x - y + z = 1\}$.
3. $F_3 = \{(x_1, 0, x_3) \in \mathbb{R}^3 \mid x_1 \leqslant 0 \text{ et } x_3 \in \mathbb{R}\}$.

Exercise 3 *Show that in the space* $\mathbb{R}^3$, *the vectors* $u = (-5, 6, 4)$, $v = (1, 0, -2)$ *and* $w = (0, 3, 5)$ *are linearly independent.*

Exercise 4 *let* F *be the subset of* $\mathbb{R}^3$ *defined as :*

$$F = \{(x_1, x_2, 0) \in \mathbb{R}^3 \mid x_1, x_2 \in \mathbb{R}\}$$

1. *Show that* F *is a vector subspace of* $\mathbb{R}^3$.
2. *Find the basis of* F. *Deduce the dimension of* F.

Exercise 5 *Let* $\mathbb{R}^3$ *be the vector space on the field* $\mathbb{R}$,

1. *Show that the set* $\{v_1 = (0, 1, 1), v_2 = (1, 0, 1), v_3 = (1, 1, 0)\}$ *is a basis of* $\mathbb{R}^3$.
2. *Find the components of a vector* $w = (1, 1, 1)$ *in the basis* $\{v_1, v_2, v_3\}$