BATNA 2 UNIVERSITY - MOSTFA BENBOULAID -
MATHEMATICS AND COMPUTER SCIENCE FACULTY
STATISTICS AND DATA SCIENCE DEPARTMENT

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TUTORIAL SERIES 1–INFINITE SERIES – ANALYSIS 3

Exercise 1. Which series converge, and which diverge ? Give reasons for your answer. If a series converges find its sum

$$a)\ \sum_{n\geq 1}\frac{4}{(4n-3)(4n+1)},\quad b)\ \sum_{n\geq 1}\frac{2^n}{5^{n-1}},\quad c)\ \sum_{n\geq 1}\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}},\quad d)\ \sum_{n\geq 1}\frac{e^{2n}}{6^{n-1}},$$

$$e)\ \sum_{n\geq 0}\frac{3}{(3n+1)(3n+4)},\quad f)\ \sum_{n\geq 2}\ln(1-\frac{1}{n^2}),\quad g)\ \sum_{n\geq 0}\frac{1}{\sqrt{n+2}+\sqrt{n+1}}.$$

Exercise 2. Decide whether the following series converge or diverge :

$$1)\ \sum_{n\geq 0}\frac{3n+4}{5n+1},\quad 2)\ \sum_{n\geq 1}\frac{1}{\sqrt{n}},\quad 3)\ \sum_{n\geq 1}\arctan\left(\frac{1}{2n^2}\right),$$

$$4)\ \sum_{n\geq 1}e^{\sin\left(\frac{1}{n(n+1)}\right)},\quad 5)\ \sum_{n\geq 0}\frac{n+\sqrt{n}}{2n^3+1},\quad 6)\ \sum_{n\geq 1}\frac{1}{n}-\frac{1}{n^2}.$$

$$7)\ \sum_{n\geq 1}\frac{n^2+2}{n^2},\quad 8)\ \sum_{n\geq 1}\frac{3}{\sqrt{n}},\quad 9)\ \sum_{n\geq 1}\frac{1}{\sqrt{n(n+1)}}.$$

Exercise 3. Using the Limit Comparison Test, tell whether each series converges or diverges (For some of them, simple comparison work)

$$1)\ \sum_{n\geq 1}\frac{n^3}{4n^4+n^2},\quad 2)\ \sum_{n\geq 1}\frac{\ln n}{\sqrt{n^3+n-1}},\quad 3)\ \sum_{n\geq 1}\frac{1}{\sqrt{n^2+n}},\quad 4)\ \sum_{n\geq 0}\frac{\sqrt{n}}{n^2+1},$$

$$5)\ \sum_{n\geq 1}\sin\frac{1}{n^2},\quad 6)\ \sum_{n\geq 1}\frac{1}{n+\sqrt{n}},\quad 7)\ \sum_{n\geq 1}\frac{2^n+n}{2^n.n},\quad 8)\ \sum_{n\geq 2}\frac{n^2}{n^4-1},$$

$$9)\ \sum_{n\geq 1}\frac{1+\ln n}{n\sqrt{n}},\quad 10)\ \sum_{n\geq 1}\sqrt{n}\ln\left(1+\frac{1}{n^2}\right).$$

Exercise 4. Using the Ratio's Test, the Root's Test or the Limit Comparison Test, test for convergence the infinite series whose nth term is :

$$1)\ u_n=\left(\frac{n^2+1}{2n^2+1}\right)^n,\quad 2)\ u_n=\frac{2^n}{1.3.5...........(2n-1)},\quad 3)\ u_n=\frac{\ln n+(\ln n)^3}{n^3+\sqrt{n}},$$

$$4)\ u_n=\frac{(n+3)!}{3!.n!.3^n},\quad 5)\ u_n=\frac{2^n}{n!},\quad 6)\ u_n=\left(1+\frac{\alpha}{n}\right)^{-n^2}\ (\alpha\in\mathbb{R}),\ ,$$

$$7)\ u_n=\frac{\ln(n!)}{n!},\quad 8)\ u_n=\frac{n+3}{n(\ln n)^2+3},\quad 9)\ u_n=n!\prod_{k=1}^{n}\sin\left(\frac{1}{2^k}\right),$$

$$10)\ u_n=\sum_{n\geq 0}\frac{(n!)^2}{(2n)!},\quad 11)\ u_n=\frac{n}{2^n}\quad 12)\ u_n=\frac{(\ln n)^n}{n^n},$$

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Exercise 5. Test for convergence the infinite series whose nth term is :

1) $u_n = \dfrac{1.3.5......(2n-1)}{2.4.6......(2n)}$, 2) $u_n = \dfrac{n!\ln n}{n(n+2)!}$, 3) $u_n = \dfrac{n!}{e^n}$,

4) $u_n = \dfrac{n!}{n^n}$, 5) $u_n = \dfrac{n2^n(n+1)!}{3^n n!}$, 6) $u_n = \dfrac{1.3.5......(2n-1)}{2.4.6......(2n)(2n+2)}$,

7) $u_n = \dfrac{1.3.5..........(2n-1)}{4^n 2^n n!}$, 8) $u_n = \dfrac{n}{e^n}$, 9) $u_n = \dfrac{a^n}{(1+a)(1+a^2)...(1+a^n)}(a>0)$.

Exercise 6. It is given that : $\sum\limits_{n=0}^{+\infty} \dfrac{1}{n!} = e$.

By using this fact alone find the exact value of each of the sums $\sum\limits_{n=0}^{+\infty} \dfrac{n+1}{n!}$ and $\sum\limits_{n=0}^{+\infty} \dfrac{n^2-2}{n!}$.

Exercise 7. Determine whether the given series is absolutely convergent, conditionally convergent or divergent

1) $\sum\limits_{n\geq 1} \dfrac{(-1)^n}{n}$, 2) $\sum\limits_{n\geq 1} \dfrac{\cos(n\pi)}{\sqrt{n}}$, 3) $\sum\limits_{n\geq 1} \dfrac{(-1)^n}{n^2}$, 4) $\sum\limits_{n\geq 0} \dfrac{(-1)^n}{\sqrt{n^2+1}}$,

5) $\sum\limits_{n\geq 1} (-1)^n \dfrac{n^3}{3^n}$, 6) $\sum\limits_{n\geq 1} \dfrac{5^n}{n}$.

Exercise 8. Using the Dirichlet's Test, tell whether each series converges or diverges

1) $\sum\limits_{n\geq 1} \dfrac{e^{i2n}}{n}$, 2) $\sum\limits_{n\geq 2} \dfrac{\sin(3n)}{\ln n}$, 3) $\sum\limits_{n\geq 2} \dfrac{\cos(n)}{n\ln n}$.

Exercise 9.(Additional)

Suppose that $\sum\limits_{n\geqslant 0} u_n$ and $\sum\limits_{n\geqslant 0} v_n$ are series with nonnegative terms. Show the following statements :

— If the series $\sum u_n$ is convergent, then $\sum u_n^2$ is also convergent .

— The series $\sum u_n$ and $\sum \dfrac{u_n}{1+u_n}$, both converge or both diverge.

— If the series $\sum u_n$ and $\sum v_n$ are convergent, then $\sum \sqrt{u_n.v_n}$ and $\sum \max(u_n, v_n)$ are convergent.

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