

# Input-Output Linearization of an Induction Motor Using MRAS Observer

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## Abstract

*This paper presents the non-linear control of an induction motor (IM). The objective of nonlinear control is to can control separately flux and the speed, several techniques of control are used for (IM), The technique of control oriented flux (FOC) whichpermits the decoupling between input and output variables, so (IM) is assimilate to continuous current motor , this method has a problem is how exactly oriented the axis d on the flux .However, feedback linearization amounts to cancelling the nonlinearities in a nonlinear system so that the closed- loop (CL) dynamics is in a linear form .*

*A goal of feedback linearizationis to can controlled separately flux and the speed ,the motor model is strongly nonlinear then it's composed to the autonomous and mono-variables too under systems so every under system presented an independence loop of control for each variables is given .*

*We proposed the MRAS (system adaptive reference model) for speed and rotor fluxobserver.*

**Keywords:** *input-output modeling, induction motor, LIES derivates, PWM inverter, MRAS observer*

## 1. Introduction

Feedback linearization amounts to cancelling the nonlinearities in a nonlinear system so that the closed- loop (CL) dynamics is in a linear form.

A goal of nonlinear control is to can controlled separately flux and the speed, the motor model is strongly nonlinear then it's composed to the autonomous and mono-variables too under systems, so every under system presented an independence loop of control for each variables is given.

Then the induction motors constitute a theoretically interesting and practically important class of non-linear systems. The control task is further complicated by the fact that induction motors are subject to unknown load disturbances and change in values of parameters during its operation. The control engineering community is faced then with the challenging problem of controlling a highly nonlinear system, with varying parameters, where the regulated outputs, besides some of them being not measurable, are perturbed by an unknown disturbance signal [1-3].

The major of the control's strategies necessities know of the precise mechanic speed. There is a lot of inconvenient of employment the speed captor's then we will exploited the estimators and observers [13-14].We combined the MRAS observer with the input-output linearization control, to verify the behaviour of the control numerical simulations are performed for different operating conditions.

The paper is organized as follows: in Section 2, we give the mathematical input-output model for the induction motor by differentiating the outputs with Lie derivative [5-6] and expressing all states an inputs in terms of these outputs [7], in Section 3, we represented the feedback linearization of IM and in the Section 4 we give the control of flux and speed of linear system, in the five section we give the model mathematic of the observer finally we give the results simulation and the conclusion.

Simulation was given by the classic PWM.

## 2. Model of the Induction Motor

The state equations in the stationary reference frame of an induction motor can be writing as [8-9]:

$$\begin{aligned} \dot{X} &= F(X) + GU \\ Y &= H(X) \end{aligned} \quad (1)$$

With  $X = [i_{\alpha\beta} \ i_{s\alpha} \ i_{s\beta} \ \phi_{r\alpha} \ \phi_{r\beta} \ \Omega]^T$

$$F(x) = \begin{bmatrix} -\gamma i_{s\alpha} + \frac{K}{T_r} + p\Omega K \phi_{r\beta} \\ -\gamma i_{s\beta} - p\Omega K \phi_{r\alpha} + \frac{K}{T_r} \phi_{r\beta} \\ \frac{M}{T_r} i_{s\alpha} - \frac{1}{T_r} \phi_{r\alpha} - p\Omega \phi_{r\beta} \\ \frac{M}{T_r} i_{s\beta} + p\Omega \phi_{r\alpha} - \frac{1}{T_r} \phi_{r\beta} \\ p \frac{M}{JL_r} (\phi_{r\alpha} i_{s\beta} - \phi_{r\beta} i_{s\alpha}) - \frac{1}{J} (C_r + f\Omega) \end{bmatrix} \quad G = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sigma L_s} & 0 & 0 & 0 \end{bmatrix} U$$

$$= [u_{s\alpha} \ u_{s\beta}]^T$$

$$K = \frac{M}{\sigma L_s L_r}, \sigma = 1 - \frac{M^2}{L_s L_r}, \gamma = \frac{R_s}{\sigma L_s} + \frac{R_r M^2}{\sigma L_s L_r^2}$$

$R_s, R_r$ : Stator and rotor resistances

$L_s, L_r$ : Stator and rotor inductances

$M$ : Mutual inductance

$\Omega$ : Motor speed.

$\phi_r$ : Rotor flux norm.

The variables, which are controlled, are the flux  $\phi_r$  and the speed  $\Omega$ .

$$Y(X) = \begin{bmatrix} y_1(X) \\ y_2(X) \end{bmatrix} = \begin{bmatrix} h_1(X) \\ h_2(X) \end{bmatrix} = \begin{bmatrix} x_3^2 + x_4^2 \\ \Omega \end{bmatrix} = \begin{bmatrix} \phi_r^2 \\ \Omega \end{bmatrix} \quad (2)$$

## 3. Feedback Linearization of IM

### 3.1. Relative Degree of the Flux

$$h_1(x) = (\phi_{r\alpha}^2 + \phi_{r\beta}^2) \quad (3)$$

$$l_f h_1 = \frac{2}{T_r} [M(\phi_{r\alpha} i_{s\beta} + \phi_{r\beta} i_{s\alpha}) - (\phi_{r\alpha}^2 + \phi_{r\beta}^2)] \quad (4)$$

$$L_{g1} L_f h_1 = 2R_r K \phi_{r\alpha} \quad (5)$$

$$L_f^2 h_1 = \left(\frac{4}{T_r^2} + \frac{2K}{T_r^2} M\right) (\phi_{r\alpha}^2 + \phi_{r\beta}^2) - \left(\frac{6M}{T_r^2} + \frac{2\gamma M}{T_r}\right) (\phi_{r\alpha}^2 i_{s\alpha} + \phi_{r\beta}^2 i_{s\beta}) + \frac{2Mp\Omega}{T_r} (\phi_{r\alpha}^2 i_{s\beta} - \phi_{r\beta}^2 i_{s\alpha}) + 2\frac{M^2}{T_r^2} (i_{s\alpha} + i_{s\beta}) \quad (6)$$

$$L_{g2} L_f h_1 = 2R_r K \phi_{r\beta} \quad (7)$$

The degree of  $h_1(x)$  is  $r_1=2$ .

### 3.2. Relative Degree of Speed

$$h_2(x) = \Omega \quad (8)$$

$$h_2 = \frac{-pM}{JL_r} (\phi_{r\alpha} i_{s\beta} - \phi_{r\beta} i_{s\alpha}) - \frac{1}{j} (C_r - f\Omega) \quad (9)$$

$$L_{g1} L_f h_2 = -p \frac{K}{j} \phi_{r\beta}$$

$$(10) L_f^2 h_2 = \frac{-pM}{JL_r} \left[ \left( \frac{1}{T_r} + \gamma + \frac{f}{j} \right) (\phi_{r\alpha}^2 i_{s\beta} - \phi_{r\beta}^2 i_{s\alpha}) + p\Omega (\phi_{r\alpha} i_{s\alpha} - \phi_{r\beta} i_{s\beta}) \right] - \frac{f}{j^2} (C_r - f\Omega) + p\Omega K (\phi_{r\alpha}^2 - \phi_{r\beta}^2) \quad (11)$$

$$L_{g2} L_f h_2 = p \frac{K}{j} \phi_{r\alpha} \quad (12)$$

The degree of  $h_2(x)$  is  $r_2=2$ .

### 3.3. Global Relative Degree

The global relative degree is lower than the order  $n$  of the system  $r = r_1 + r_2 = 4 < n = 5$ . The system is siding partly linear zed; we obtain a not observable dynamics of order 2. [11] Define the change of coordinates:

$$\left\{ \begin{array}{l} z_1 = h_1(x) \\ z_2 = L_f h_1(x) \\ z_3 = h_2(x) \\ z_4 = \arctan\left(\frac{\phi_{r\beta}}{\phi_{r\alpha}}\right) \\ z_5 = \Omega \end{array} \right. \quad (13)$$

The derivatives of the outputs are given in the new coordinate system by:

$$\left\{ \begin{array}{l} \dot{z}_1 = z_2 \\ \dot{z}_2 = v_1 \\ \dot{z}_3 = v_2 \\ \dot{z}_4 = p\Omega + \frac{R_r z_2}{p z_1} \\ \dot{z}_5 = -\frac{1}{j} (z_2 - C_r - f z_5) \end{array} \right. \quad (14)$$

### 3.4. Decoupling Matrix

The matrix defines a relation between the input (U) and the output (Y(X)) is giving by the Expression (15).

$$\begin{bmatrix} \frac{d\phi_r^2}{dt} \\ \frac{d\Omega}{dt} \end{bmatrix} = A(X) + D(X) \begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \end{bmatrix} \quad (15)$$

Where

$$A(X) = [L_f^2 h_1 L_f h_2]^T$$

The decoupling matrix is:

$$D(X) = \begin{bmatrix} Lg_1 L_f h_1 & Lg_2 L_f h_2 \\ Lg_1 L_f h_2 & Lg_2 L_f h_2 \end{bmatrix}$$

$$\text{and } \det D = \frac{2pR_r KM}{J\sigma L_s L_r} (\phi_{\alpha r}^2 + \phi_{r\beta}^2) \neq 0$$

The nonlinear feedback provide to the system a linear compoment input/output

$$\begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \end{bmatrix} = D(X)^{-1} \left[ -A(X) + \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \right] \quad (14)$$

Where

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \ddot{Y}_1(X) \\ \ddot{Y}_2(X) \end{bmatrix} = \begin{bmatrix} \frac{d\phi_r^2}{dt} \\ \frac{d\Omega}{dt} \end{bmatrix}$$

#### 4. Control Flux and Speed of Linear System

The internal outputs ( $V_1, V_2$ ) are definite:

$$V_1 = \frac{d^2\phi_r^2}{dt} = -K_{11}(\phi_r^2 - \phi_{ref}^2) - K_{12} \left( \frac{d}{dt}\phi_r^2 - \frac{d}{dt}\phi_{ref}^2 \right) + \frac{d^2\phi_{ref}^2}{dt} \quad (15)$$

$$V_2 = \frac{d^2\Omega}{dt} = -K_{22}(\Omega - \Omega_{ref}) - K_{12} \left( \frac{d}{dt}\Omega - \frac{d}{dt}\Omega_{ref} \right) + \frac{d^2\Omega_{ref}}{dt} \quad (16)$$

The error of the track in (CL) is:

$$\ddot{e}_1 + K_{12}\dot{e}_1 + K_{11}e_1 = 0 \quad (17)$$

$$\ddot{e}_2 + K_{21}\dot{e}_2 + K_{22}e_2 = 0 \quad (18)$$

With:

$$e_1 = \phi_r^2 - \phi_{ref}^2$$

$$e_{21} = \Omega - \Omega_{ref}$$

The coefficients  $K_{11}, K_{12}, K_{21}, K_{22}$  are choosing to satisfy asymptotic stability and excellent tracking

$$V_1 = -K_{11}(\phi_r^2 - \phi_{ref}^2) - K_{12} \frac{d}{dt}\phi_r^2 \quad (19)$$

$$V_2 = -K_{22}(\Omega - \Omega_{ref}) - K_{21} \frac{d}{dt}\Omega \quad (20)$$

$$\begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \end{bmatrix} = D(X)^{-1} \left[ -A(X) + \left( -K_{11}e_1 - K_{12} \frac{d}{dt}\phi_r^2 - K_{22}e_2 - K_{21} \left( \frac{C_e}{J} - \frac{1}{J}(C_r + f\Omega) \right) \right) \right] \quad (21)$$

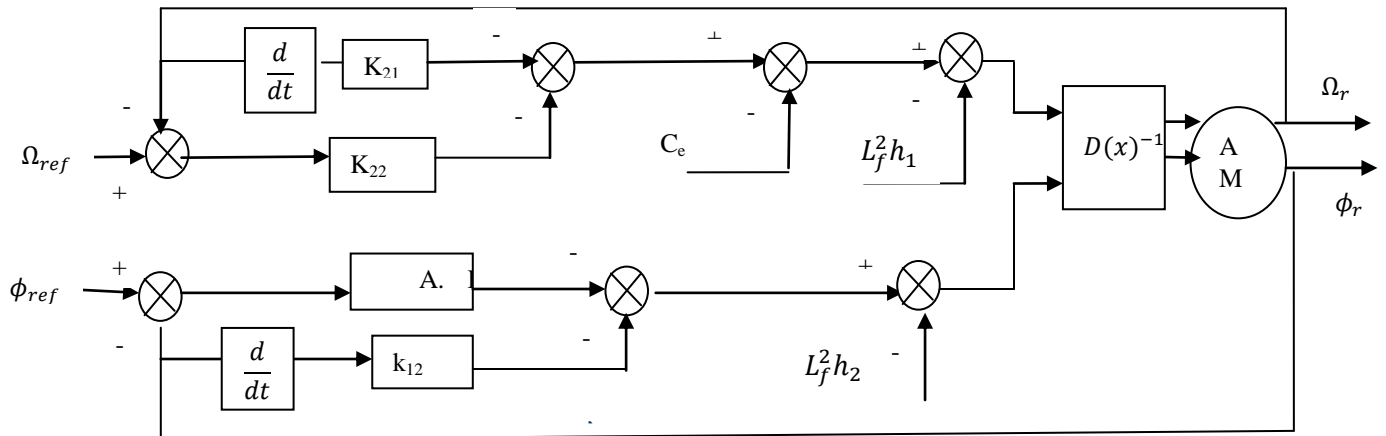


Figure 1. Schema Block of Non-Linear Control

## 5. MRAS Observer

The proposed observer is constituted by two model [10-12-13]:

### 5.1. Reference Model

$$\begin{cases} \frac{d}{dt} = \frac{L_r}{L_m} (V_{\alpha S} - R_S I_{\alpha S} - \sigma L_S \frac{dI_{\alpha S}}{dt}) \\ \frac{d}{dt} = \frac{L_r}{L_m} (V_{\beta S} - R_S I_{\beta S} - \sigma L_S \frac{dI_{\beta S}}{dt}) \end{cases} \quad (22)$$

### 5.2. Adaptive Model

$$\begin{cases} \frac{d\hat{\phi}_{\alpha r}}{dt} = \frac{-1}{T_r} \hat{\phi}_{\alpha r} - p\hat{\Omega}\hat{\phi}_{\beta r} + \frac{L_m}{T_r} I_{\alpha S} \\ \frac{d\hat{\phi}_{\beta r}}{dt} = \frac{-1}{T_r} \hat{\phi}_{\beta r} - p\hat{\Omega}\hat{\phi}_{\alpha r} + \frac{L_m}{T_r} I_{\beta S} \end{cases} \quad (23)$$

The error is definite by:

$$e = \hat{\phi}_{\alpha r} \hat{\phi}_{\beta r} - \hat{\phi}_{\alpha r} \hat{\phi}_{\beta r} \quad (24)$$

Adaptation mechanism

The input of the mechanism is activating by the error:

$$\frac{d\hat{\phi}}{dt} - \frac{d\hat{\phi}}{dt} = \frac{de}{dt} = \left( \frac{-R_r}{L_r} + jw \right) \hat{\phi} - \left( \frac{-R_r}{L_r} + j\hat{w} \right) \hat{\phi} = \left( \frac{-R_r}{L_r} + jw \right) (\hat{\phi} - \hat{\phi}) + j(w - \hat{w}) \hat{\phi} \quad (25)$$

Then we have

$$\begin{bmatrix} \frac{de_{\alpha}}{dt} \\ \frac{de_{\beta}}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-1}{T_r} & -w \\ w & \frac{-1}{T_r} \end{bmatrix} \begin{bmatrix} e_{\alpha} \\ e_{\beta} \end{bmatrix} + (w - \hat{w}) \begin{bmatrix} \hat{\phi}_{\alpha r} \\ \hat{\phi}_{\beta r} \end{bmatrix}$$

And  $\hat{w} = k_p \Delta e + k_i \int_0^t \Delta e dx$

Next

$$\frac{\Delta w(s)}{\Delta e(s)} = k_p + \frac{k_i}{s}$$



Figure 2. MRAS Observer

6. Simulation Results

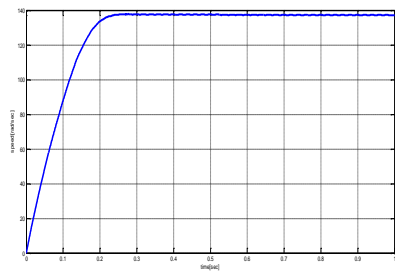


Figure 3. Speed Rotor [rad/sec]

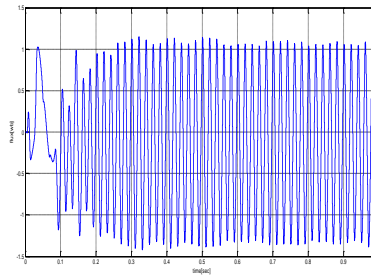


Figure 4. Rotor Flux [Wb]

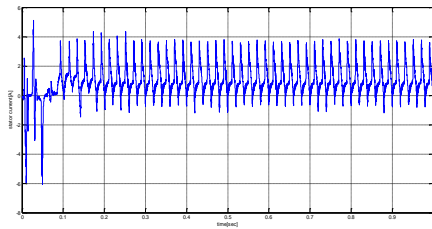


Figure 5. Stator Current [A]

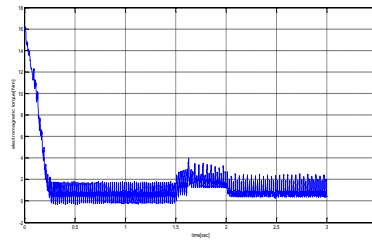


Figure 6. Electromagnetic Torque [Nm]

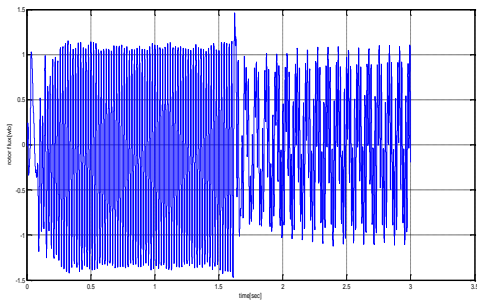


Figure 7. Rotor Flux [Wb]

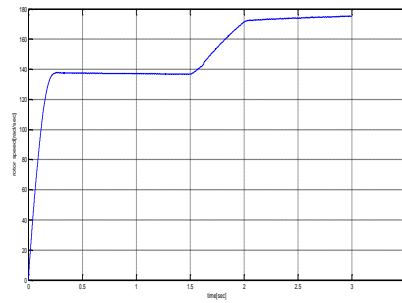
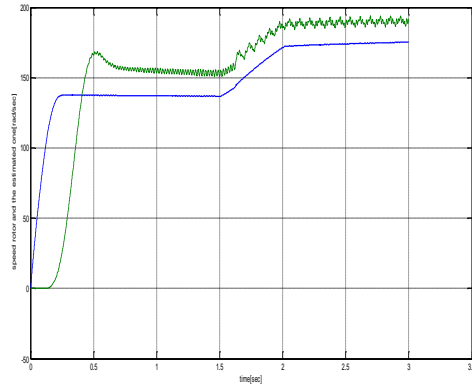
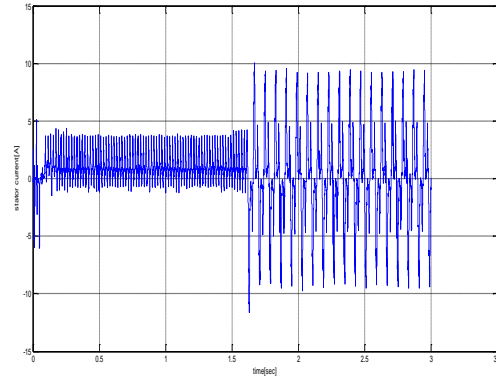


Figure 8. Rotor Speed [rad/sec]



**Figure 9. Rotor Speed and the Estimatedone [rad/sec]**



**Figure 10. Stator Current [A]**

## 7. Simulations

Input-output linearization gives us the possibility that we can control separately the speed and the flux,

The results are showing in Figure (3-10) represented the simulation, so response of speed and the test of the variation of the speed given in Figure 3, the observer gives us the possibility to estimate the speed but with a small delay in the response of the observer and undulation in the Figure 9, the norm of flux remnants to 1 [Wb] Figure 4 in the company of reduction in this value if the speed change in Figure 7 and the current greater than before Figure 10, the regulators of the speed and the flux gives us the choice to control their intensity separately after application the regulation of Hurwitz the simulation is given by MLI inverter.

## 8. Conclusion

The non-linear control gives a good tracking for the speed with basing of its static and dynamic properties. The results show that the decoupling between the parameters of IM is excellent. This technique gives a better amelioration for the performances of system and the MRAS observer give the estimated speed in the future we will change the classic regulators.

### Specifications of the Induction Motor

1.1KW, 220/380V, 50Hz, 1500 rpm

### Parameters of the Induction Motor

$R_r = 3.6 \Omega$ ,  $J = 0.015 \text{ Kg m}^2$ ,

$R_s = 8.0 \Omega$ ,  $f = 0.005 \text{ Nm s}$

$L_r = 0.47 \text{ H}$ ,  $P = 2$

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