Course :
Reinforced Concrete I

3rd year civil engineering license

Dr. ZENDAOUI Abdelhakim

INTRODUCTION TO REINFORCED CONCRETE

Reinforced concrete is a composite material made up of two basic components: concrete and steel. These materials are combined in a way that allows for the most economical use of the strength of each

We call reinforced concrete the material obtained by adding steel bars to concrete. These steel bars are generally called **reinforcements**.

In the combination of concrete and steel, concrete resists the compressive forces while steel resists tensile stresses, taking over any compressive stresses that the concrete cannot support.

$$\text{Concrete}\begin{cases}\nearrow \text{Compressive strength} = 20 \text{ MPa à } 40 \text{ MPa}\\ \searrow \text{Tensile strength} = 2 \text{ MPa à } 4\text{MPa}\end{cases}$$

Steel $\rightarrow$ Tensile or Compressive strength = 200 MPa à 500 MPa

<u>Advantages and disadvantages of reinforced concrete :</u>

<u>1- Advantages :</u>

a. Economic interest : Concrete is the most economical material for resisting compressive loads and can be combined with other elements.

Steel is currently the only material used for reinforcement manufacturing because it is the cheapest and has the ability to resist tensile stress.

b. Flexibility of use : the concrete being placed in molds (formwork) like a paste; it is possible to create constructions with a wide variety of shapes and the reinforcements can be easily integrated, and the connections between concrete components are made by simple contact

Therefore, reinforced concrete is easily processed for prefabrication in factories.

c. Maintenance savings : Reinforced concrete constructions require no maintenance compared to steel structures, which are susceptible to corrosion and require periodic repainting.

d. Fire resistance : Reinforced concrete buildings perform much better in fires than metal or wooden structures. Concrete's poor thermal conductivity delays the effects of heat on the reinforcement, allowing for the restoration of the structure after superficial repairs, which is not possible with metal or wooden structures. This property made it possible to use reinforced concrete in certain parts of the furnaces.

e. Resistance to accidental forces : Reinforced concrete, due to its significant weight, exhibits greater resistance to changes in load compared to other construction methods.

f. Durability : Reinforced concrete is a durable material, that can resist a variety of attacks and stresses over its design life. The only condition is the protection of the reinforcement.

2. Disadvantages :

a. The weight : Reinforced concrete structures are heavier than other construction methods.

b. Execution : To build a reinforced concrete structure, it is necessary to :

- Formwork preparation that requires a lot of time and significant structural work. This formwork must remain in place until the concrete reaches sufficient strength.

- the placement of the reinforcements..

- During and after concrete placement, precautions must be taken to protect it from freezing and water evaporation.

- Quality control of the material improved during mixing.

c. Brutality of accidents : Failures in reinforced concrete structures are generally sudden and catastrophic, often caused by design or construction mistakes.

d. Difficulty in modifying an existing structure : It is difficult to modify an element already made.

CHAPTER 01: COMPONENTS OF REINFORCED CONCRETE

1- Concrete :

1.1- Generality :

Concrete is a monolithic material obtained by combining cement, water, and aggregates (sand, gravel, cobbles).

The proportions of these components and the cement content, expressed by the water-cement ratio (W/C), are the primary factors determining the required properties and qualities of concrete before and after hardening.

Fresh concrete must be workable so that it can perfectly fill the mold provided without segregation, and stable in service, capable of supporting the imposed loads.

The desired qualities of good concrete :

- High mechanical resistance (25-40 MPa).
- Impermeability to water and absence of chemical reaction with steel.
- Good implementation (easy to pour).
- Good resistance over time.

These qualities will be obtained by playing on the following parameters :

- The quality of the cement and aggregates.
- The dosage (quantity).
- A good mixture (homogeneity).

1.2- The components of a concrete :

- Aggregates : Rock fragments of various sizes are classified into three categories based on their dimensions, ranging from 0.08mm to 100mm. These categories include sands, gravels, and cobbles.

- Cement : Is a binder consisting of a fine powder which, when mixed with water, becomes a more or less thick paste, hardens under the sole influence of water.

- Mixing water : The amount of water used for mixing is greater than that required for hydrating the cement because some of the water evaporates when the concrete hardens in air.

- Admixtures : Concrete admixtures are chemicals products that are added to concrete (in very small quantities) before or after mixing in order to modify certain properties of fresh concrete, concrete during hardening or hardened concrete.

1.3- Physical and mechanical properties of concrete :

1.3.1- Density :

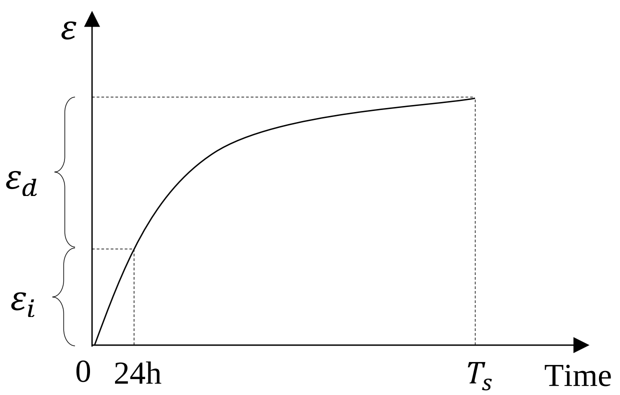
- The density of typical concretes ranges from 2200 to 2400 kg/m^3 ($2200 \leq \zeta \leq 2400$ kg/m^3).
- The density of lightweight aggregate concrete $\rightarrow 700 \leq \zeta \leq 1500$ kg/m^3.
- The density of heavy aggregate concrete $\rightarrow 3500 \leq \zeta \leq 4000$ kg/m^3.
- The density of reinforced concrete $\rightarrow \zeta = 2500$ kg/m^3.

1.3.2- Deferred strains of concrete :

a) Shrinkage : It is the shortening of unloaded concrete during hardening, it is due to the evaporation of excess mixing water not combined with the cement. Shrinkage can cause deformations of the order of 0.2 to 0.5 mm per meter.

Shrinkage increases with decreasing element thickness and increasing cement content. Additionally, shrinkage is more pronounced in drier climates.

b) Creep : It is the progressive increase in relative strains over time under constant loads. Creep increases with increasing water content. It decreases with increasing cement content and the age of the specimen at testing.



ε_i : Instantaneous deformation under applied loads.

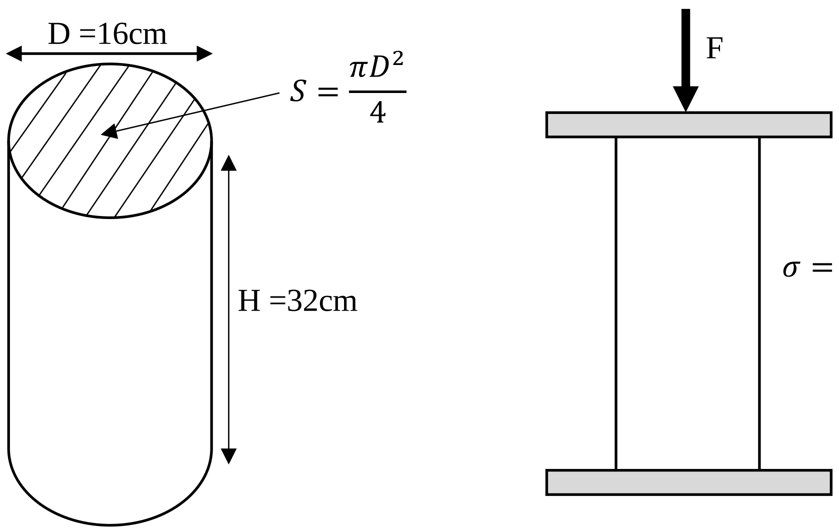
ε_d : Deferred deformation under constant load.

$$\varepsilon_d = 2\,\varepsilon_i$$

1.3.3- Concrete strains under short actions (duration < 24 hours) :

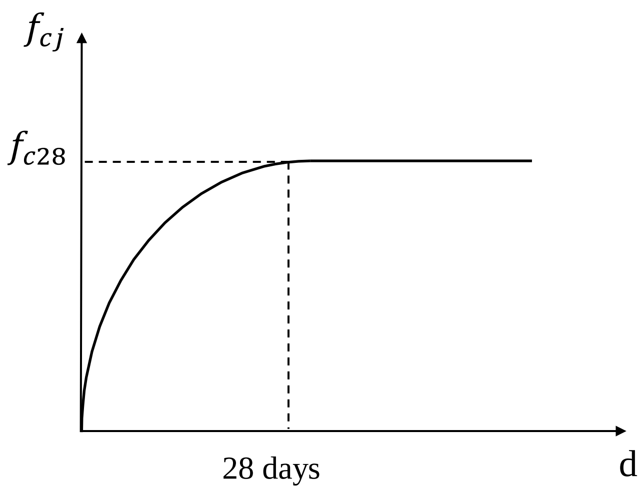
a) Compressive strength : Concrete is defined by the value of its compressive strength at 28 days of age, known as the required (specified) characteristic value.

The compression test is carried out on concrete cylinders as follows :



$$\sigma = \frac{F}{S} = \frac{F.4}{\pi D^2} \ (N/mm^2 \ ou \ MPa)$$

The following curve shows the evolution of the compressive strength of concrete over time:

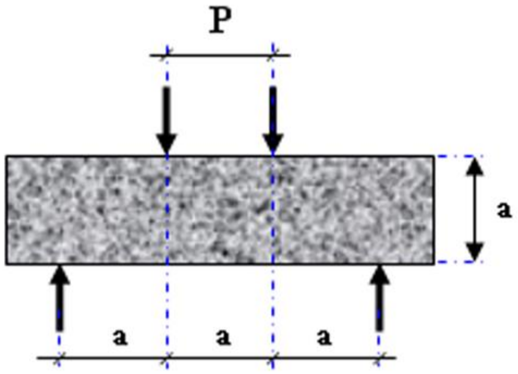


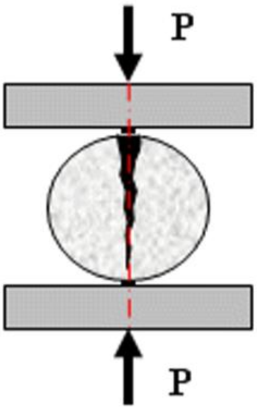
- f_{cj} : The compressive strength at 'j' day.
- f_{c28} : The compressive strength at '28' days; This is the characteristic strength of concrete.

b) Tensile strength : The characteristic tensile strength of concrete at 'd' days noted and conventionally defined by the relation :

$$f_{tj} = 0{,}6 + 0{,}06 f_{cj}$$

The tensile strength of concrete can also be measured through flexural or splitting tensile tests, as illustrated in the following figures."





Flexural test

Splitting test

The tensile strength of concrete is significantly lower than its compressive strength, typically with a ratio of $\frac{1}{12}$ to $\frac{1}{13}$ for ordinary concrete. As a result, it is often neglected in the design and calculation of reinforced concrete structures."

1.3.4- Longitudinal strains of concrete :

a) Elastic Modulus of Concrete (Young's modulus) : Under normal stresses, we distinguish between instantaneous Young's modulus (E_{ij}) and the deferred Young's modulus (E_{vj}), the first is used for calculations under instantaneous loads (loading time <24H), and for long-term loads we use the deferred Young's modulus which takes into account the creep strains of the concrete.

- The instantaneous elastic modulus : $E_{ij} = 11000(f_{cj})^{1/3}$ with : $f_{cj} = f_{c28} \leq 60MPa$

- The deferred elastic modulus is taken ${}^{1}/{}_{3}$ of the instantaneous module :

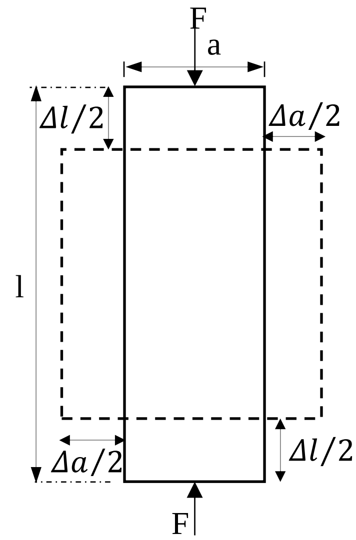
$$E_{vj} = \frac{1}{3}\, E_{id} = 3700(f_{cj})^{1/3}$$

b) Poisson's ratio : When a concrete specimen of length l is subjected to compressive forces, not only a longitudinal shortening Δl occurs but also a transverse swelling. If a is the initial dimension of the side of the experiment, this dimension becomes a+Δa after swelling.

We call the Poisson's ratio : $\upsilon = \frac{\Delta a/a}{\Delta l/l}$

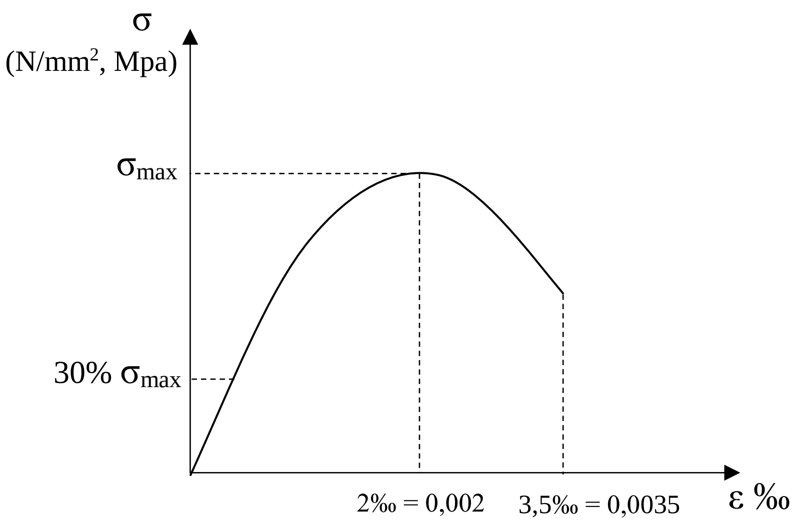
- $\upsilon = 0{,}2$; When the concrete is not cracked, i.e. at Serviceability Limit State (S.L.S.).

  $\upsilon = 0$; Otherwise, i.e. at Ultimate Limit State (E.L.U.)



1.3.5- Stress-strain diagram of concrete :

a) Experimental (real) diagram : When a concrete specimen is subjected to a compression test, the behavior curve (σ, ε) has the following form :

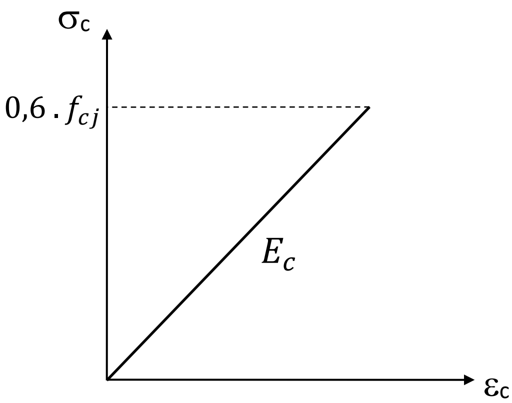


For concrete used in practice, please note the following :

- Up to 30% of the maximum stress σ_{max} , the curve (σ, ε) is linear.
- At maximum stress σ_{max} , strain $\varepsilon = 2‰ = 0{,}002$.
- Concrete begins to disintegrate when deformation exceeds 3.5‰.

b) Design stress-strain diagrams :

b.1- Serviceability limit state (S.L.S.) model : The concrete is assumed to remain in the elastic range (no cracks), and the following linear elastic diagram is therefore adopted:

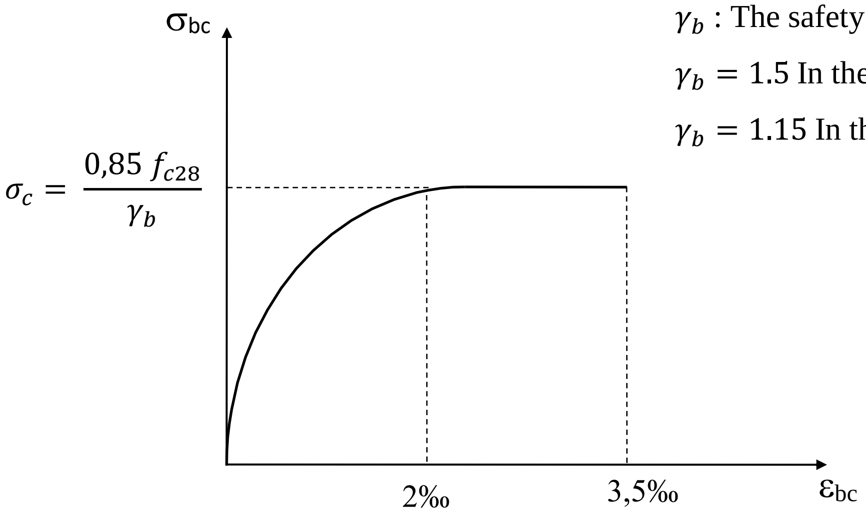


The limiting stress of concrete at S.L.S. is : :

$\overline{\sigma_c} = 0{,}6 \,.\, f_{c28}$

E_c : Elastic Modulus of Concrete

b.2- Ultimate limit state (U.L.S) model : For U.L.S. calculations, the real behavior of concrete is modeled by the 'parabola-rectangle' law, which is represented in the following diagram:



γ_b : The safety factor of concrete

$\gamma_b = 1.5$ In the general case (vertical loads).

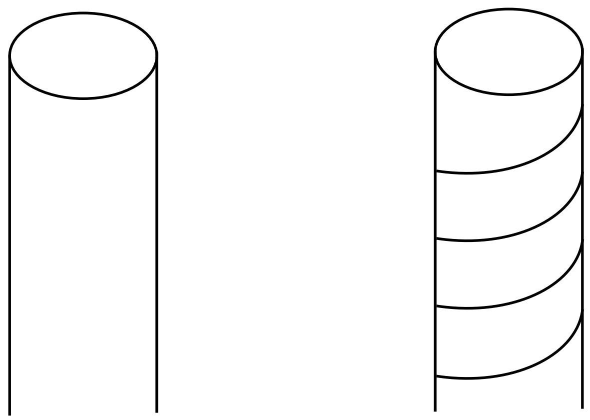
$\gamma_b = 1.15$ In the accidental case (earthquake).

The coefficient '0.85' is intended to take account of long-duration processing and loading.

2- Steel (Reinforcements) :

The reinforcements must comply with the regulatory texts, the mechanical character which serves as a basis for the justifications is the **guaranteed yield strength** designated by f_e. The longitudinal elastic modulus of the steel is taken equal to : $E_s = 200000\ MPa$.

Reinforcement for reinforced concrete consists of steel rebars differentiated by their surface profile and grade: plain bars (AD) and high bond bars or deformed bars (HA) are commonly used.



plain bars (AD) high bond (HA)

- For plain bars, there are two grades, FeE215 and FeE235, with guaranteed yield strengths of 215 and 235 MPa.

- For high-bond bars, the grades are FeE400 and FeE500, corresponding to guaranteed yield strengths of 400 and 500 MPa.

For example: FeE400 $\Rightarrow \; f_e = 400MPa$

La longueur de la barre d'armature (AD ou AH) est entre 6m et 12m, il existe différents diamètres : 5 – 6 – 8 – 10 – 12 – 14 – 16 – 20 – 25 – 32 - 40 mm.

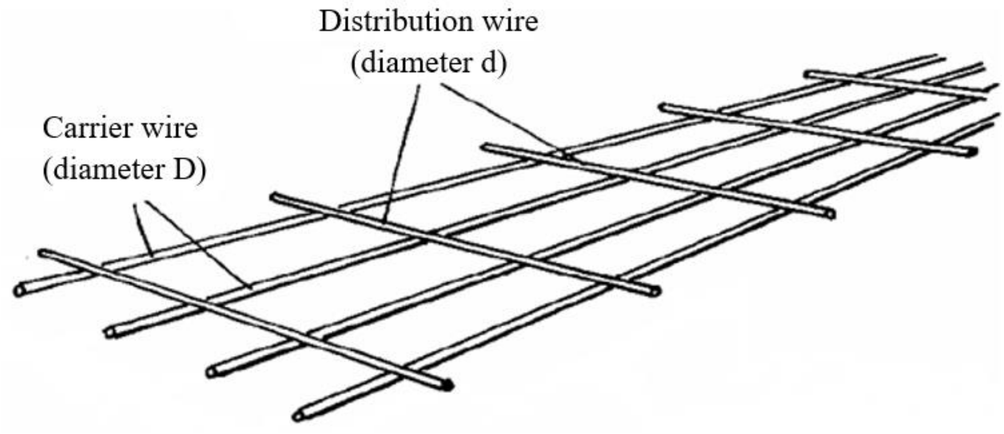
The length of the reinforcement rebars (AD or AH) is between 6m and 12m, and different diameters are available: 5 - 6 - 8 - 10 - 12 - 14 - 16 - 20 - 25 - 32 - 40 mm.

(reinforcing mesh) : Some reinforced concrete elements, such as slabs and shear walls, are reinforced in two perpendicular directions using reinforcing mesh. This reinforcement consists of a grid of welded wire, typically composed of larger diameter longitudinal wires to carry the primary tensile forces and smaller diameter transverse wires to distribute the stresses more uniformly. The orthogonal arrangement of the reinforcement provides resistance to both bending and shear forces.

Commonly used diameters are: 3 - 3,5 - 4 - 4,5 - 5 - 6 - 7 - 8 - 9 - 10 - 12 mm.

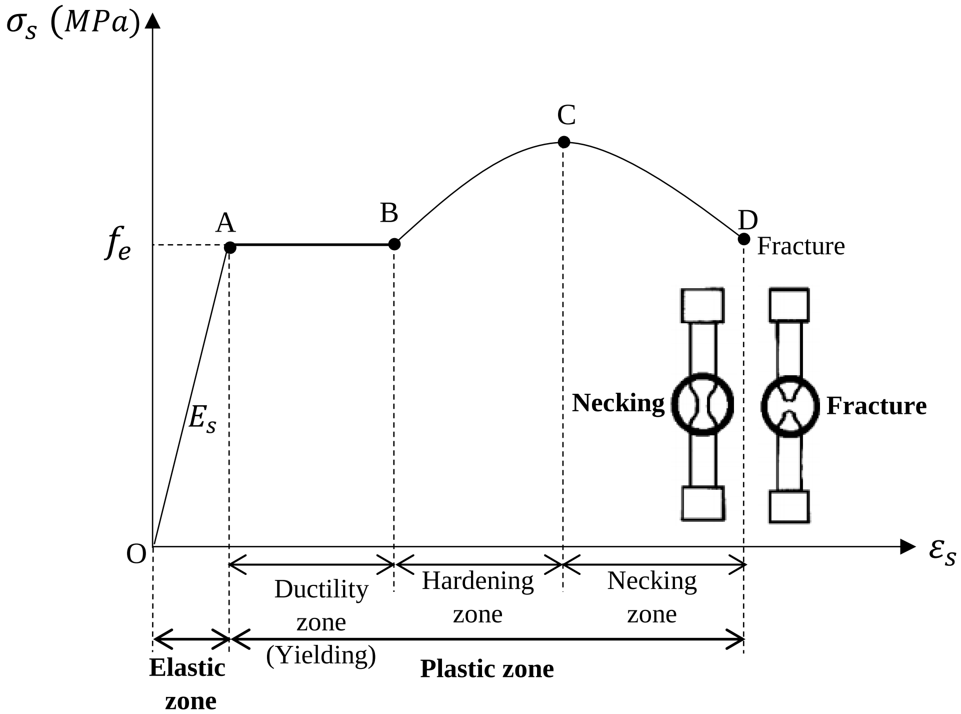
The spacing between the carrier wires : 50 - 75 - 100 - 125 - 150 - 200 mm.

The spacing between distribution wires : 100 - 150 - 200 - 250 -300 mm.



2.1- Stress-strain diagram for steel :

a) Experimental (real) diagram : A steel bar is loaded to failure in uniaxial tension to determine its mechanical characteristics. A stress-strain diagram is obtained as follows :



The curve is divided into several zones:

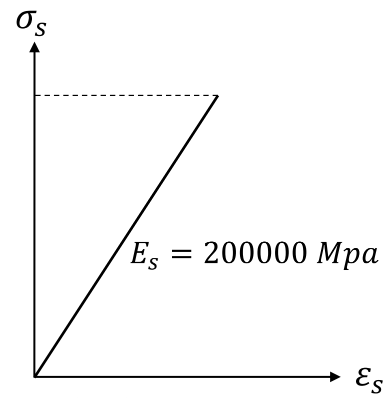
- Zone OA : This is a straight line for which there is proportionality between strain and applied stress according to HOOK's law ($\sigma_s = E_S\ \varepsilon_s$). This is a reversible elastic zone.
- Zone AB : This is a horizontal bearing called the ductility bearing, translating into elongation under constant load, it is the plastic zone.
- Zone BC : The load increases again with elongation up to point C. If the specimen is unloaded in this plastic zone, remanent elongation is observed. This is the strain-hardening zone.

- Zone CD : Elongation continues, despite decreasing load, up to point D where failure occurs. In this zone, necking occurs: plastic deformation is localized in a small portion of the specimen and is therefore no longer homogeneous.

b) Design stress-strain diagrams:

b.1- Serviceability limit state (S.L.S.) : It is assumed that the steel rebars work in the elastic range, so the following linear elastic diagram is adopted:

To calculate the steel stress σ_s at S.L.S., it will be limited only in the limited state of crack opening:



1- No-damaging cracking: no need to check.

2- Damaging cracking : $\sigma_s = min\left\{\frac{2}{3} f_e\ ; 110\sqrt{\mu\ f_{t28}}\right\}$

3- Highly damaging cracking : $\sigma_s = min\left\{\frac{1}{2} f_e\ ; 90\sqrt{\mu\ f_{t28}}\right\}$

μ : Coefficient de fissuration $\Rightarrow \mu = 1$ pour les ronds lisses (AD).

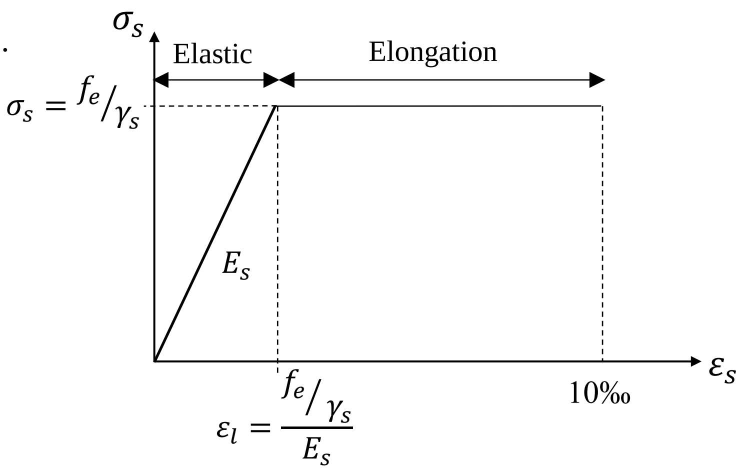
$\mu = 1.6$ Pour les barres à haute adhérence (AH).

b.2- Ultimate limit state (U.L.S) : The behavior of reinforcement in the U.L.S. calculations is based on a perfect elasto-pastic law, as described in the following stress-strain diagram:

γ_s : The safety factor of steel

$\gamma_s = 1.15$ In the general case (vertical loads).

$\gamma_s = 1$ In the accidental case (earthquake).



SECTIONS REELLES D'ARMATURES

Section en cm² de N armatures de diamètre Ø en mm

Ø :	5	6	8	10	12	14	16	20	25	32	40
1	0,20	0,28	0,50	0,79	1,13	1,54	2,01	3,14	4,91	8,04	12,57
2	0,39	0,57	1,01	1,57	2,26	3,08	4,02	6,28	9,82	16,08	25,13
3	0,59	0,85	1,51	2,36	3,39	4,62	6,03	9,42	14,73	24,13	37,70
4	0,79	1,13	2,01	3,14	4,52	6,16	8,04	12,57	19,64	32,17	50,27
5	0,98	1,41	2,51	3,93	5,65	7,70	10,05	15,71	24,54	40,21	62,83
6	1.18	1,70	3,02	4,71	6,79	9,24	12,06	18,85	29,45	48,25	75,40
7	1,37	1,98	3,52	5,50	7,92	10,78	14,07	21,99	34,36	56,30	87,96
8	1,57	2,26	4,02	6,28	9,05	12,32	16,08	25,13	39,27	64,34	100,53
9	1,77	2,54	4,52	7,07	10,18	13,85	18,10	28,27	44,18	72,38	113,10
10	1,96	2,83	5,03	7,85	11,31	15,39	20,11	31,42	49,09	80,42	125,66
11	2,16	3,11	5,53	8,64	12,44	16,93	22,12	34,56	54,00	88,47	138,23
12	2,36	3,39	6,03	9,42	13,57	18,47	24,13	37,70	58,91	96,51	150,8
13	2,55	3,68	6,53	10,21	14,70	20,01	26,14	40,84	63,81	104,55	163,36
14	2,75	3,96	7,04	11,00	15,83	21,55	28,15	43,98	68,72	112,59	175,93
15	2,95	4,24	7,54	11,78	16,96	23,09	30,16	47,12	73,63	120,64	188,5
16	3,14	4,52	8,04	12,57	18,10	24,63	32,17	50,27	78,54	128,68	201,06
17	3,34	4,81	8,55	13,35	19,23	26,17	34,18	53,41	83,45	136,72	213,63
18	3,53	5,09	9,05	14,14	20,36	27,71	36,19	56,55	88,36	144,76	226,2
19	3,73	5,37	9,55	14,92	21,49	29,25	38,20	59,69	93,27	152,81	238,76
20	3,93	5,65	10,05	15,71	22,62	30,79	40,21	62,83	98,17	160,85	251,33

3- Actions and loadings :

3.1- Actions :

Actions are the forces applied to the deformations imposed on a structure. There are three types of actions :

- Dead loads or permanent loads.
- Variable loads or live loads.
- Accidental loads

a) Dead loads (G) : These are continuous actions whose intensity is constant over time. Example : the structure's own weight, weight of fixed equipment, etc.).

b) live loads (Q) : These are actions whose intensity varies frequently and significantly over time. The duration of application is very short compared to the lifespan of constructions :

- Operating charges.
- Climatic charges (snow and wind).
- Temperature effects.

c) Accidental actions (FA) : These are actions arising from phenomena that occur rarely and have a short duration of application, for example: wind, earthquake...

3.2- Loadings :

These are Normal forces, shear forces, bending moments, and torsional moments, are calculated from the applied loads using the principles of mechanics of materials.

3.3- Action combinations :

To determine the loads, the following combinations of actions are used :

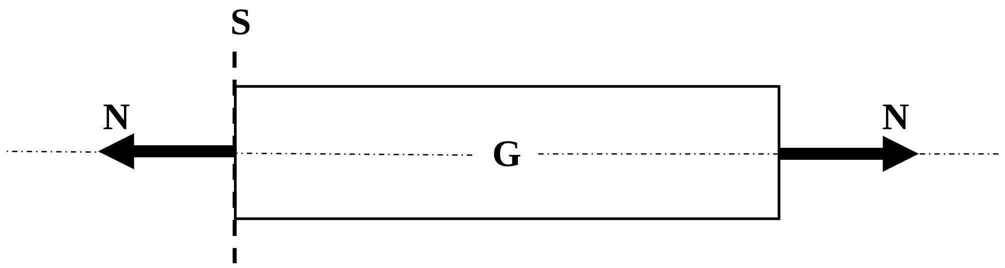
1- at the Ultimate limit state (U.L.S) : $\mathrm{q_u} = 1{,}35\ \mathrm{G} + 1{,}5\ \mathrm{Q}$

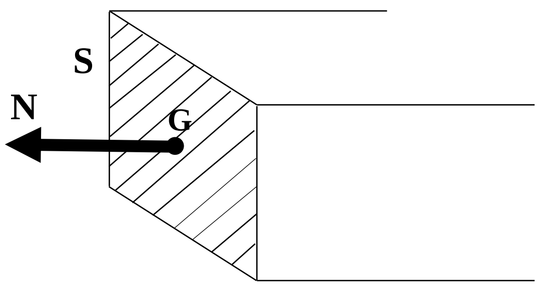
2- at the Serviceability limit state (S.L.S.) : $\mathrm{q}_s = \mathrm{G} + \mathrm{Q}$

CHAPTER 02 : SIMPLE TENSION

1- Definition :

A member is subjected to simple traction when the forces acting on it, and located on the same side relative to a straight section (S) can be reduced to a single traction force N, perpendicular to the section (S) and parallel to the mean line, and passing through the center of gravity G.





2- Determining longitudinal reinforcement :

2.1- Ultimate limit state (U.L.S.) : Given that, for safety reasons, tensile concrete is neglected, it follows that longitudinal reinforcement of total cross-section A must be able to balance the applied tensile force N on its own.

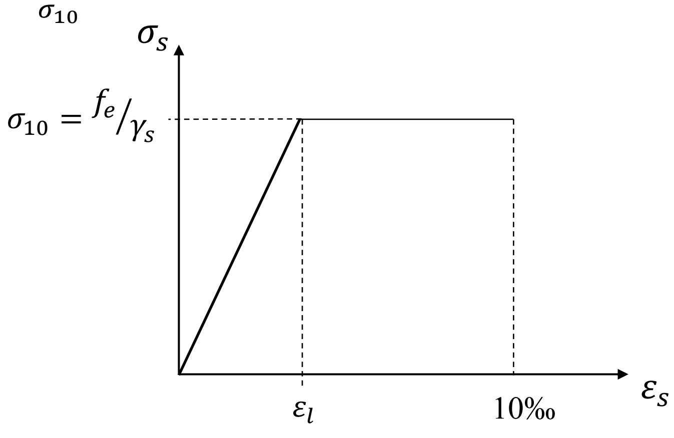
Tge reinforcement cross-section at U.L.S is : $A_{su} = \frac{N_u}{\sigma_{10}}$

With : Nu : Tensile force at U.L.S.

$\sigma_{10} = \frac{f_e}{\gamma_s}$

$\gamma_s = 1.15$ (General case)

$\gamma_s = 1$ (Accidental case)



2.2- At Serviceability Limit State (S.L.S.): The reinforcement cross-section at S.L.S. is :

$$A_{ss} = \frac{N_s}{\sigma_s}$$

With : Ns : Tensile force at S.L.S..

σ_s : Steel stress at S.L.S. is a function of cracking :

1- Non-damaging cracking: no need to check.

2- Damaging cracking : $\sigma_s = min\left\{\frac{2}{3}f_e\ ; 110\sqrt{\mu\ f_{t28}}\right\}$

3- Highly damaging cracking : $\sigma_s = min\left\{\frac{1}{2}f_e\ ; 90\sqrt{\mu\ f_{t28}}\right\}$

2.3- Non-brittle condition : The tensioned section is considered to be non-brittle if the reinforcements are working at their elastic limit (f_e) can balance the stresses causing concrete cracking in this section.

The force causing cracking in concrete : $N = B\,.f_{t28}$

B : The concrete section

f_{t28} : Tensile strength of concrete

Therefore, the non-brittle condition must be verified as follows :

$$A_{sB} \geq \frac{N}{f_e} \Rightarrow A_{sB} \geq \frac{B\ f_{t28}}{f_e}$$

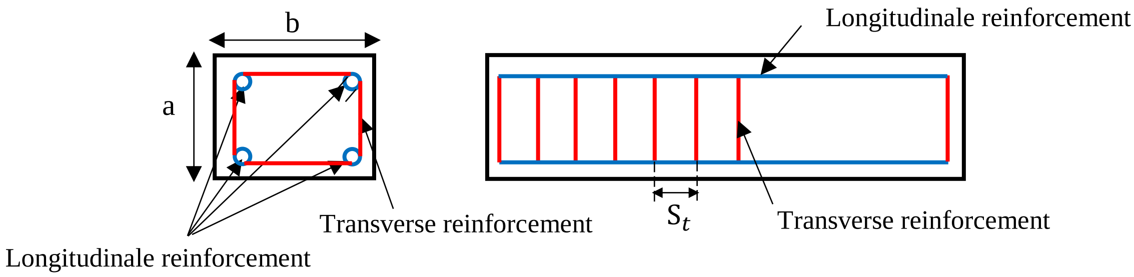
The cross-section of longitudinal reinforcement required to resist tensile force N is :

$$A_s = Max\{A_{su}\ ; A_{ss}\ ;\ A_{sB}\}$$

3- Transverse reinforcement :

They play no part in tensile strength. Their diameter is calculated as follows : $\phi_t = 0{,}3\ \phi_L$ with : $\phi_{t\ min} = 6mm$.

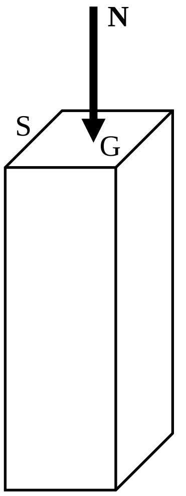
Spacing : $S_t = \min\left(40\text{ cm}\ ;\ a + 10\text{ cm}\right)$ with, a : the smallest dimension.



CHAPTER 03 : SIMPLE COMPRESSION

1- Definition :

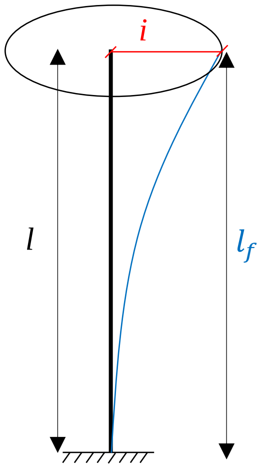
An element is subjected to simple compression when it is reduced to a normal compressive force N applied to the center of gravity of the straight section S.



2- Slenderness:

Slenderness λ is the ratio between the buckling length l_f and the radius of gyration i :

$$\lambda = \frac{l_f}{i}$$



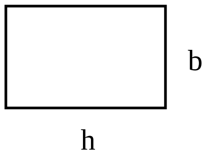
$$l \neq l_f$$

- **Definition of radius of gyration i :**

$$i = \sqrt{\frac{I}{S}}$$

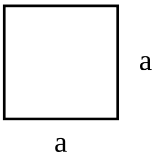
I : The moment of inertia along the small dimension.

S : Cross-sectional area of the column.

- For a rectangular section: 

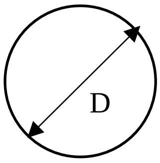
Moment of inertia: $I = \frac{h.b^3}{12}$

$$\Rightarrow i = \sqrt{\frac{I}{S}} = \sqrt{\frac{\frac{h.b^3}{12}}{b.h}} \Rightarrow i = \frac{b}{3{,}46}$$

- For a square section: 

: Moment of inertia: $I = \frac{a^4}{12}$

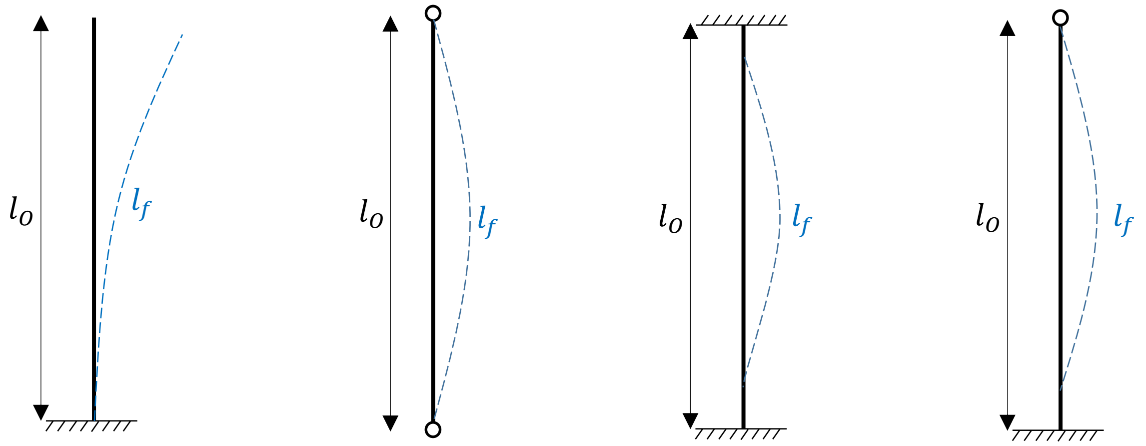
$$\Rightarrow i = \sqrt{\frac{I}{S}} = \sqrt{\frac{\frac{a^4}{12}}{a^2}} \Rightarrow i = \frac{a}{3{,}46}$$

- For a circular section: 

: Moment of inertia: $I = \frac{\pi.D^4}{64}$

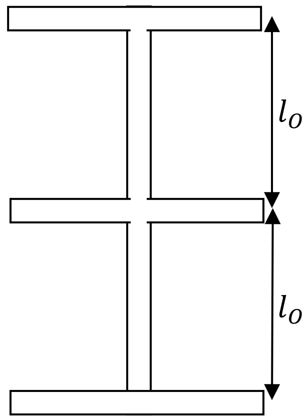
$$\Rightarrow i = \sqrt{\frac{I}{S}} = \sqrt{\frac{\frac{\pi.D^4}{64}}{\frac{\pi.D^2}{4}}} \Rightarrow i = \frac{D}{4}$$

- **Definition of buckling length l_f :** Dependent on element length (lo) and the type of connection.



$l_f = 2l_O$ $\quad$ $l_f = l_O$ $\quad$ $l_f = 0.5l_O$ $\quad$ $l_f = \frac{l_O}{\sqrt{2}}$

❖ $\boldsymbol{l_f = 0.7 l_0}$: If the column is embedded in a foundation block or in the case of columns between multiple storeys.



So the slenderness calculation :

- For a rectangular section ⇒ $\lambda = 3{,}46\frac{l_f}{b}$, With b : is the smallest side.
- For a square section ⇒ $\lambda = 3{,}46\frac{l_f}{a}$
- For a circular section ⇒ $\lambda = 4\frac{l_f}{D}$, Avec D : le diamètre.

3- Determining the resisting force :

$$N_u = (N_b + N_a)\alpha$$

N_u : Compressive force at U.L.S

N_b : The force of concrete

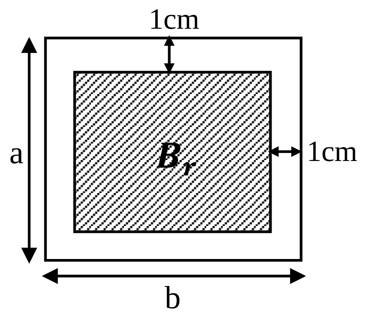
N_a : The force of steel

α : Multiplication Coefficient

$$N_b = \frac{B_r\ f_{c28}}{0{,}9\,\gamma_b}$$

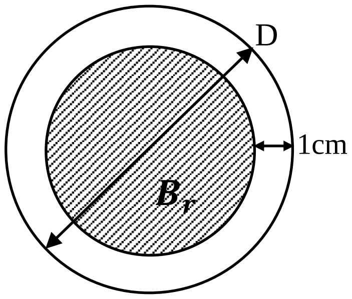
B_r : Reduced concrete cross-section: for greater safety, the cross-section is reduced by eliminating 1cm on each side.

- For a rectangular section :



$$\Rightarrow \quad B_r = (a - 2cm)(b - 2cm)$$

- For a circular section :



$$\Rightarrow \qquad B_r = \frac{\pi(D-2cm)^2}{4}$$

$$N_a = \frac{A_s\, f_e}{\gamma_s}$$

$$\Rightarrow\ N_u = \left[\frac{B_r\ f_{c28}}{0{,}9\,\gamma_b} + \frac{A_s\, f_e}{\gamma_s}\right]\alpha$$

With : $\alpha = \dfrac{0{,}85}{1+0{,}2\left(\frac{\lambda}{35}\right)^2}$; Si $\lambda \leq 50$

$$\alpha = 0{,}6\left(\frac{50}{\lambda}\right)^2 \ ; \quad \text{Si } \lambda > 50$$

4- Determining reinforcement :

4.1- Longitudinal reinforcement:

<u>a) Form stability limit state :</u> The form stability section is derived from the force expression N_u :

$$N_u = \left[\frac{B_r\ f_{c28}}{0{,}9\,\gamma_b} + \frac{A_s\, f_e}{\gamma_s}\right]\alpha$$

$$\Rightarrow A_{sf} = \left[\frac{N_u}{\alpha} - \frac{B_r\ f_{c28}}{0{,}9\,\gamma_b}\right]\frac{\gamma_s}{f_e}$$

<u>b) Ultimate strength :</u> The reinforcement is arranged parallel to the neutral axis of the part. In simple compression, the concrete and steel stresses are :

$$\sigma_{bc} = \frac{0{,}85\, f_{c28}}{\gamma_b}\ et\ \sigma_s = \frac{f_e}{\gamma_s}$$

$$N_u = N_b + N_a$$

$$N_u = B\ \sigma_{bc} + A_s\ \sigma_s$$

$$N_u = B\ \frac{0{,}85\ f_{c28}}{\gamma_b} + A_s \frac{f_e}{\gamma_s}$$

$$\Rightarrow A_{sR} = \left[N_u - \frac{B\ 0{,}85\ f_{c28}}{\gamma_b}\right] \frac{\gamma_s}{f_e}$$

c) Serviceability limit state :

$$A_{ss} = \frac{1}{15}\left[\frac{N_s}{\sigma_b} - B\right] \qquad With:\ \sigma_b = 0{,}6\ f_{c28}$$

d) Minimum section :

$$A_{s\,min} = max\left[\frac{4(b+h)}{100}\ ;\ \frac{0{,}2\ b\ h}{100}\right]$$

To obtain the required reinforcement section, take the maximum value of the four sections calculated above::

$$A_s = Max\{A_{sf}\ ; A_{sR}\ ;\ A_{ss}\ ; A_{s\,min}\}$$

4.1- Transverse reinforcement:

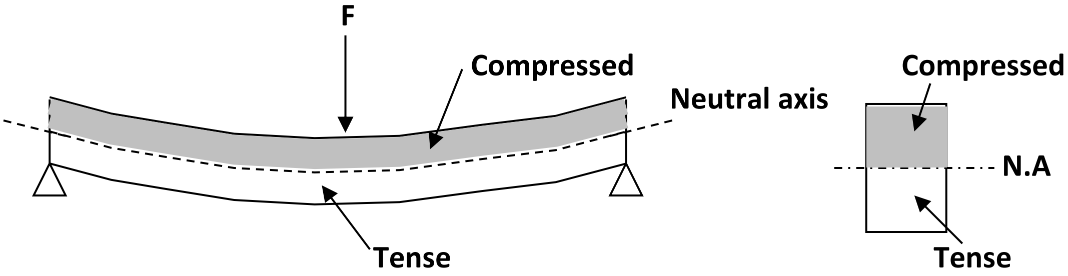
Their main role is to prevent buckling of the longitudinal reinforcement. To calculate transverse reinforcement, we use the same method as for simple tension.

$\phi_t = 0{,}3\ \phi_L$ And spacing : $S_t = \min(40\ \text{cm}\ ;\ a + 10\ \text{cm})$

CHAPTER 04 : SIMPLE BENDING

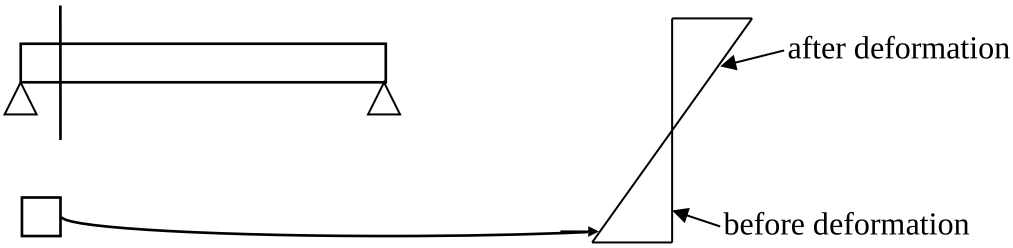
1- Definition :

A beam is loaded in simple bending when it is subjected to the action of forces arranged symmetrically in relation to the center of gravity.

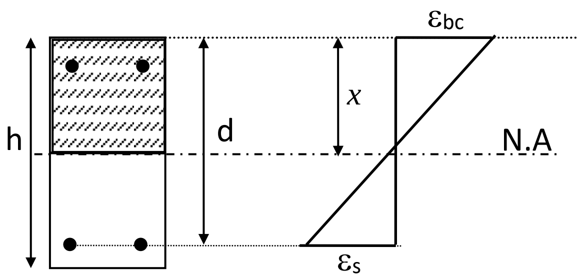


2- Calculation assumptions at U.L.S :

***Assumption* (1) :** Any plane section before deformation remains flat after deformation.



***Assumption* (2) :** There is no relative slip between concrete and steel. It follows from this assumption that fiber deformations are proportional to their distance from the neutral axis.



ε_{bc} : deformation of concrete under compression..

ε_{s} : deformation of tensioned steel..

x : la distance de l'axe neutre.

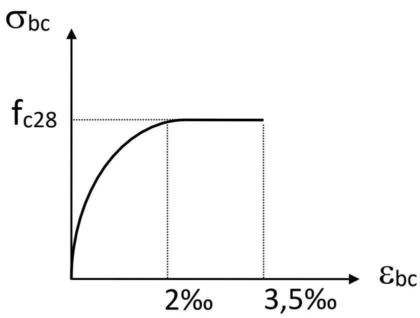
d : distance of the center of gravity of the tensioned reinforcement. If d is unknown, we take : d=0.9h

$$\frac{\varepsilon_{bc}}{\varepsilon_{s}} = \frac{x}{d-x} \quad \text{with :} \quad \alpha = \frac{x}{d} \Rightarrow x = \alpha.d$$

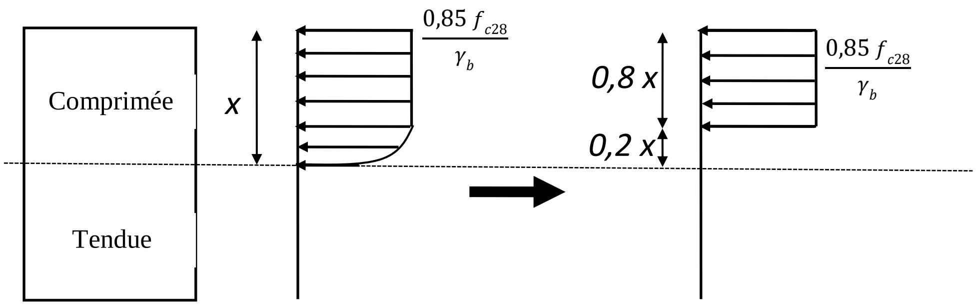
$$\varepsilon_{bc} = \frac{x}{d-x}\varepsilon_{s} = \frac{\alpha.d}{d-\alpha.d}\varepsilon_{s} \quad \Rightarrow \quad \varepsilon_{bc} = \frac{\alpha}{1-\alpha}.\varepsilon_{s} \text{ et } \varepsilon_{s} = \frac{1-\alpha}{\alpha}.\varepsilon_{bc}$$

***Assumption* (3) :** The strength of the tensioned concrete is neglected.

***Assumption* (4) :** We use the parabola-rectangle stress-strain diagram for concrete.



When the section is not fully compressed, the simplified rectangular diagram defined as follows can be used:



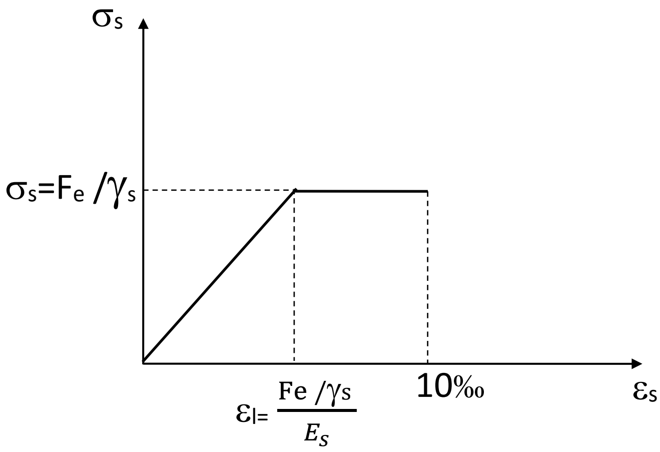
For a distance of 0,2.x from the neutral axis, stress is considered to be zero. Over the remaining distance, stress will be equal to $\frac{0{,}85\, f_{c28}}{\gamma_b}$

Assumption (5) : Steel and concrete materials are anelastic, concrete shortening is limited by 3.5‰ in compression and steel elongation will be limited to 10‰.

Assumption (6) : The steel stress-strain diagram for the calculation is as follows:

$\varepsilon_s \leq \varepsilon_l = \frac{Fe/\gamma_s}{E_s} \Rightarrow \sigma_s = E_s . \varepsilon_s$

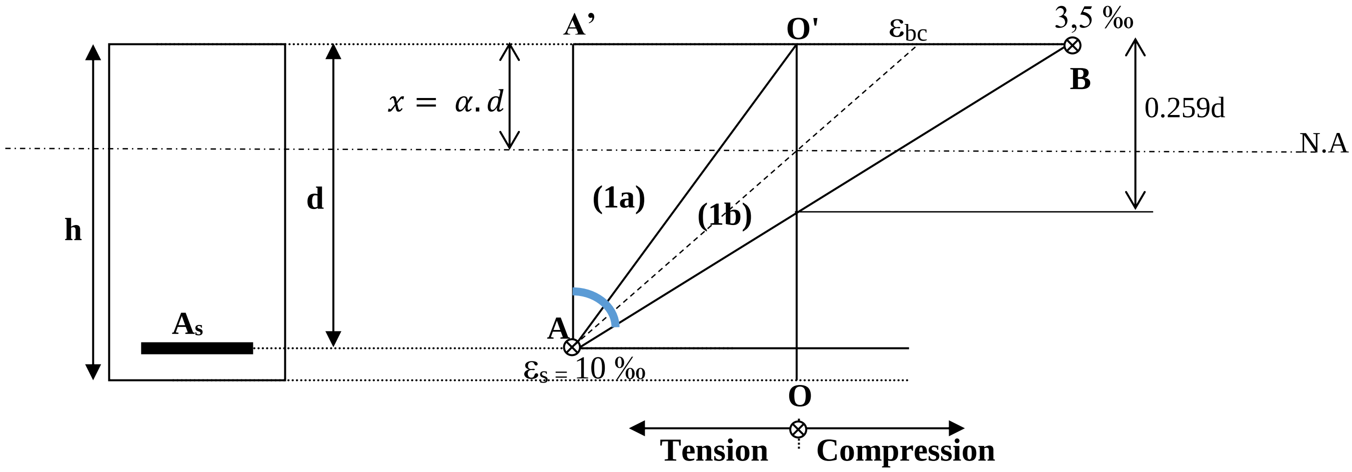
$\varepsilon_s > \varepsilon_l = \frac{Fe/\gamma_s}{E_s} \Rightarrow \sigma_s = \frac{f_e}{\gamma_s}$



3- Three-pivot method:

The Three-pivot diagram brings together the different cases that represent the deformation of the section in the ultimate limit state.

<u>Pivot A :</u> The diagram passes through pivot A corresponds to a 10‰ elongation of tensioned steel without depleting concrete strength.



This diagram will be divided into 2 areas:

Area (1a) : The concrete is always in tension, and does not contribute to the strength of the section, so this is the case of tension.

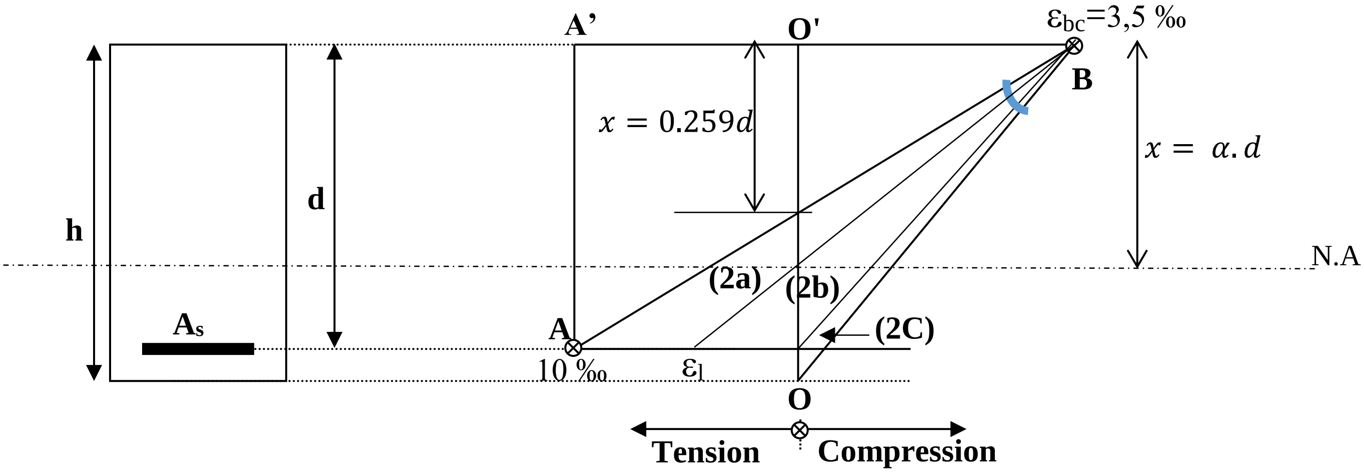
Area (1b) : The concrete is partially compressed, the shortening of the most compressed fiber remains less than 3.5 ‰.

Therefore : $0 \leq \varepsilon_{bc} \leq 3{,}5$ ‰ et $\varepsilon_s = 10$ ‰.

$$\alpha = \frac{x}{d} \leq \frac{\varepsilon_{bc}}{\varepsilon_{bc} + \varepsilon_{st}} = \frac{3{,}5}{3{,}5 + 10} = 0{,}259$$

$0 \leq \alpha \leq 0{,}259 \quad \Rightarrow \quad 0 \leq x = \alpha.d \leq 0{,}259.d$

<u>Pivot B :</u> The diagram passes through pivot B corresponds to a 3.5 ‰ shortening of the most compressed fiber of the concrete ($\varepsilon_{bc} = 3{,}5$ ‰ and $0{,}259.d < x \leq h$ and $0{,}259 < \alpha \leq 1$).



This diagram will be divided into 3 areas:

Area (2a) The elongation of the tensioned reinforcement is greater than the limit elastic elongation ε_l ; Therefore : $\varepsilon_l \leq \varepsilon_s \leq 10$ ‰ and $\sigma_s = \frac{f_e}{\gamma_s}$.

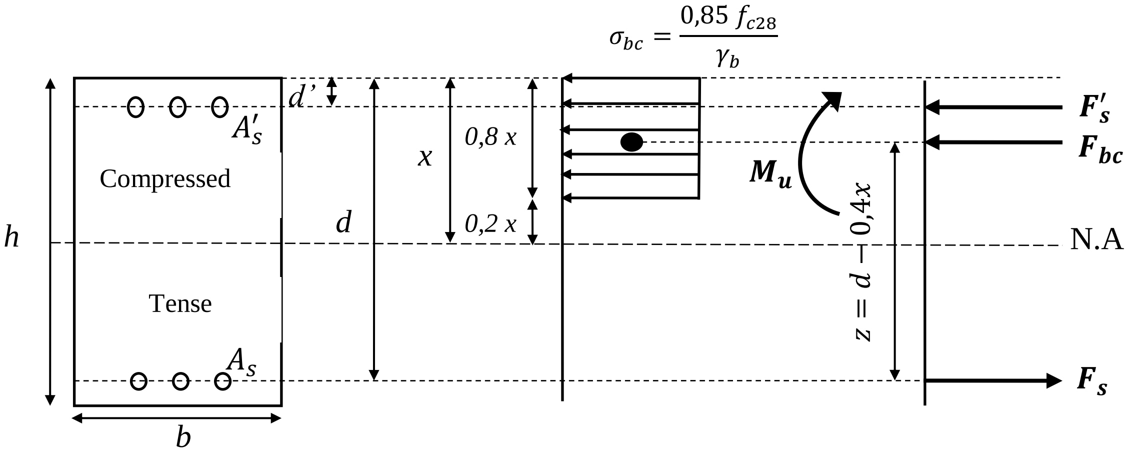
Area (2b) : The elongation of the tensioned reinforcement is less than the limit elastic elongation ε_l ; Therefore : $0 \leq \varepsilon_s \leq \varepsilon_l$ et $\sigma_s = E_s . \varepsilon_s$.

Area (2c) : Les armatures sont comprimées.

<u>Pivot C :</u> Compression case.

4- Calculation of sections in simple bending at the ultimate limit state (U.L.S):

According to the simplification of the real stress diagram of compressed concrete "parabola-rectangle" (assumption 4), the equilibrium equations for the diagram under consideration are obtained by writing that the sum of the projections of the resultants of the longitudinal forces on the axis of the element is zero, and that the sum of the moments of these resultants (expressed in relation to A_s) balances the external moment M_u :



The sum of efforts: $F_{bc} + F_s' - F_s = 0$

The sum of moments: $M_u = F_s'(d - d') + F_{bc}(d - 0{,}4x)$

The efforts will be written as follows: $F_{bc} = 0{,}8.x.b.\sigma_{bc}$

$$F_s = A_s.\sigma_s$$

$$F_s' = A_s'.\sigma_s'$$

The element is assumed to be without compressed reinforcement ($A_s' = 0$), So :

$F_{bc} - F_s = 0 \;\Rightarrow\; F_{bc} = F_s \;\Rightarrow\; 0{,}8.x.b.\sigma_{bc} = A_s.\sigma_s$

$M_u = F_{bc}(d - 0{,}4x) \;\Rightarrow\; M_u = 0{,}8.x.b.\sigma_{bc}(d - 0{,}4x)$

We have : $x = \alpha.d \;\Rightarrow\; M_u = 0{,}8.\alpha.b.d^2.\sigma_{bc}(1 - 0{,}4\alpha)$

Posing: $\eta = 0{,}8.\alpha(1 - 0{,}4\alpha)$, donc : $M_u = \eta.b.d^2.\sigma_{bc}$

We find the reduced moment $\eta = \frac{M_u}{\sigma_{bc}\, b\, d^2}$ where $\alpha = 1{,}25(1 - \sqrt{1 - 2\eta})$

According to the 3-pivot method, the stress-strain diagram shows that when the maximum deformations of compressed concrete are equal to $\varepsilon_{bc} = 3{,}5‰$ and for tensioned reinforcement $\varepsilon_s = 10‰$, the corresponding coefficients are $\alpha = 0{,}259$ et $\eta = 0{,}186$. This is the limiting case between pivots A and B.

The pivot A case : The pivot A corresponds to maximum elongation of the tensioned reinforcement ($\varepsilon_s = 10‰$) and the deformation of the compressed concrete remains variable ($0 \leq \varepsilon_{bc} \leq 3{,}5$ ‰). In this case : $0 < \alpha \leq 0{,}259$ and $\eta \leq 0{,}186$.

- If : $0{,}104 \leq \eta \leq 0{,}186$ ⇒ That is, 2 ‰ $\leq \varepsilon_{bc} \leq$ 3,5 ‰ and Simply Reinforced Section (S.R.S) ;

The sum of the moments relative to the point of application of the force F_{bc} :

$M_u = A_s.\sigma_s(d - 0{,}4x) = A_s.\sigma_s.d(1 - 0{,}4\alpha)$

$\Rightarrow A_s = \frac{M_u}{\sigma_s\, d\, \beta}$ With : $\beta = 1 - 0{,}4\alpha$ and $\sigma_s = \frac{f_e}{\gamma_s}$

If : $\eta < 0{,}104$ ⇒ The concrete is poorly used, so the section must be resized.

The pivot B case : Pivot B corresponds to maximum shortening of the compressed concrete ($\varepsilon_{bc} = 3{,}5‰$) and the deformation of the tensioned reinforcement remains variable ($0 \leq \varepsilon_s \leq 10‰$). In this case: $0{,}259 < \alpha \leq 1$ and $\eta > 0{,}186$.

The limit reduced moment is calculated : $\eta_l = 0{,}8\alpha_l(1 - 0{,}4\alpha_l)$ avec : $\alpha_l = \frac{\varepsilon_{bc}}{\varepsilon_{bc}+\varepsilon_l}$ et $\varepsilon_l = \frac{f_e}{E_s\,\gamma_s}$

- If : $\eta \geq \eta_l \Rightarrow$ The cross-section must be reinforced with tensioned A_s and compressed reinforcement A'_s (reinforcement of such a section without compressed reinforcement is not economical) $\Rightarrow$ Doubly Reinforced Section (D.R.S)

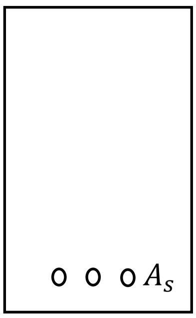
$$A_s = \frac{\eta + 0{,}8\alpha_l\,(0{,}4\alpha_l - \zeta)}{\sigma_s\,(d - d')}.\sigma_{bc}\,b\,d^2$$

$$A'_s = \frac{A_s\sigma_s - 0{,}8\alpha_l\,\sigma_{bc}\,b\,d}{\sigma_s}$$

With : $\zeta = \frac{d'}{d}$ and $\sigma_s = E_S\,\varepsilon_s$ *and* $\varepsilon_s = \frac{1-\alpha}{\alpha}\,\varepsilon_{bc}$

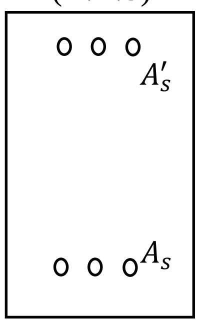
- If : $\eta < \eta_l \Rightarrow$ The section can be reinforced using tensioned reinforcement only $A_s \Rightarrow$ Simply Reinforced Section (S.R.S) $\Rightarrow$ $A_s = \frac{M_u}{\sigma_s\,d\,\beta}$

Simply Reinforced Section (S.R.S)



Only tensioned reinforcement (A_s)

Doubly Reinforced Section (D.R.S)



There are tensioned (A_s) and compressed (A'_s)