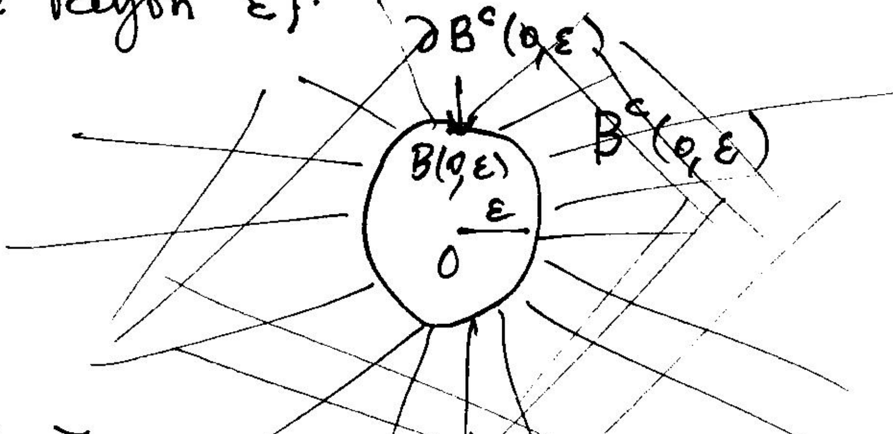
Posons $I_1 = \displaystyle\int_{\partial B^c(0,\varepsilon)} Ln(|x|) \frac{\partial \varphi}{\partial \eta_\varepsilon}(x) \, d\sigma_\varepsilon(x)$.

$$I_2 = \int_{\partial B^c(0,\varepsilon)} \varphi(x) \frac{\partial}{\partial \eta_\varepsilon}(Ln(|x|)) \, d\sigma_\varepsilon(x).$$

(Rappelons que $\partial B^c(0,\varepsilon) = C(0,\varepsilon)$ cercle de centre 0 et de rayon ε).



On traite I_1 comme suit :

$$I_1 = \int_{C(0,\varepsilon)} Ln(|x|) \frac{\partial \varphi}{\partial \vec{\eta}_\varepsilon}(x) \, d\sigma_\varepsilon(x)$$

On a

$x \in C(0,\varepsilon) \iff \frac{x}{\varepsilon} = y \in C(0,1)$, donc $|y| = 1$

Alors

$$\begin{aligned} I_1 &= \int_{C(0,1)} Ln(|\varepsilon y|) \frac{\partial \varphi}{\partial \vec{\eta}}(\varepsilon y) \, d\sigma(y) \\ &= \int_{C(0,1)} Ln(\varepsilon \underbrace{|y|}^{1}) \, \nabla\varphi(\varepsilon y)) \cdot \vec{\eta}(y) \, d\sigma(y) \\ &= \int_{C(0,1)} \varepsilon \, Ln(\varepsilon) (\nabla \varphi)(\varepsilon y) \cdot \vec{\eta}(y) \, d\sigma(y) \end{aligned}$$

La détermination du vecteur normal $\vec{\eta}$ se faite suivant la definition suivante :

$$\vec{\eta}(y) = \vec{\eta}(y_1, y_2) = \frac{\left(\frac{\partial}{\partial y_1}(\sqrt{1-y_1^2}), -1\right)}{\sqrt{1+\left(\frac{\partial}{\partial y}(\sqrt{1-y_1^2})\right)^2}}$$

Car l'équation du cercle est $y_1^2 + y_2^2 = 1$, donc

$y_2 = \sqrt{1-y_1^2} = f(y_1)$