Il vient que $\qquad (y_2 = \sqrt{1-y_1^2})$

$$\vec{\eta}(y) = \frac{\left(\frac{-y_1}{\sqrt{1-y_1^2}}, -1\right)}{\sqrt{1+\left(\frac{-y_1}{\sqrt{1-y_1^2}}\right)^2}} = \frac{(-y_1/y_2, -1)}{\sqrt{1+\frac{y_1^2}{y_2^2}}}$$

$$= (-y_1/y_2, -1) \Big/ \sqrt{(\underbrace{y_2^2+y_1^2}_{=1})/y_2^2} = (-y_1/y_2, -1) \Big/ \sqrt{\frac{1}{y_2^2}}$$

$$(N) \text{ - - - - } = (-y_1/y_2, -1)\, y_2 = (-y_1, -y_2) = -y.$$

Par conséquent

$$I_1 = \int_{C(0,1)} \varepsilon \operatorname{Ln}(\varepsilon)\, \nabla\varphi(\varepsilon y)\cdot(-y)\, d\sigma(y)$$

$$= -\varepsilon \operatorname{Ln}(\varepsilon) \int_{C(0,1)} (\nabla\varphi)(\varepsilon y)\cdot y \, d\sigma(y)$$

Comme $\nabla\varphi$ est bornée (car $\varphi \in \mathcal{D}(\mathbb{R}^2)$), alors

$$I_1 \leq \varepsilon \operatorname{Ln}\varepsilon \, \|\nabla\varphi\|_{L^\infty} \int_{C(0,1) \Leftrightarrow |y|=1} |y|\, d\sigma(y) = \varepsilon \operatorname{Ln}\varepsilon \, \|\nabla\varphi\|_{L^\infty} \int_{C(0,1)} d\sigma(y)$$

$$= 2\pi\varepsilon \operatorname{Ln}\varepsilon \, \|\nabla\varphi\|_{L^\infty} \longrightarrow 0$$

Lorsque $\varepsilon \to 0^+$

Pour le terme I_2, on a :

$$I_2 = \int_{\partial B^c(0,\varepsilon)} \varphi(x) \frac{\partial}{\partial \vec{\eta}_\varepsilon} (\operatorname{Ln}|x|)\, d\sigma_\varepsilon(x)$$

$$= \int_{C(0,\varepsilon)} \varphi(x)\, \nabla(\operatorname{Ln}|x|)\cdot \vec{\eta}_\varepsilon(x)\, d\sigma_\varepsilon(x) \Big/ \nabla \operatorname{Ln}(|x|) = \frac{x}{|x|^2}.$$

$$= \int_{C(0,\varepsilon)} \varphi(x) \frac{x}{|x|^2} \,\partial\eta_\varepsilon(x)\, d\sigma_\varepsilon(x)$$

Voir (N)

$$= \int_{C(0,1)} \varphi(\varepsilon x) \frac{\varepsilon x}{|\varepsilon x|^2}\cdot(-\varepsilon x_1, -\varepsilon x_2)\, d\sigma(x)$$

$$= -\int_{C(0,1)} \varphi(\varepsilon x) \cancel{\frac{\varepsilon^2 |x|^2}{\varepsilon^2 |x|^2}}\, d\sigma(x) \xrightarrow[\varepsilon\to 0]{TCD} 0$$