$$\int_{\mathbb{R}^N}\int_{\mathbb{R}^N} \frac{|u(x+y)-u(x)|^2}{|y|^{N+2s}}\,dx\,dy = 4\int_{\mathbb{R}^N} |\hat{u}(\xi)|^2 \int_{S_{N-1}} \int_0^{+\infty} \frac{\sin^2(\frac{1}{2} r|\xi|\cos\alpha)}{r^{1+2s}}\,dr\,d\sigma(\alpha)$$

En effectuant le ch. de varible $\eta = \frac{1}{2} r|\xi|\cos\alpha$, alors $r = 2\eta/|\xi|\cos\alpha$ et $dr = 2/|\xi|\cos\alpha \; d\eta$. Ce qui implique

$$\int_{\mathbb{R}^N}\int_{\mathbb{R}^N} \frac{|u(x+y)-u(x)|^2}{|y|^{N+2s}}\,dx\,dy = 4\int_{\mathbb{R}^N} |\hat{u}(\xi)|^2\, 2^{-2s} |\xi|^{2s} \int_{S_{N-1}} \cos^{2s}\alpha\, d\sigma(\alpha) \times \int_{\mathbb{R}^N} \frac{\sin^2\eta}{\eta^{1+2s}}\,d\eta$$

Posons $C_s = 4\, 2^{-2s} \int_{S_{N-1}} \cos^{2s}\alpha\, d\sigma(\alpha) \int_{\mathbb{R}^N} \frac{\sin^2\eta}{\eta^{1+2s}}\,d\eta < +\infty$.

Finalement, on obtient que

$$\int_{\mathbb{R}^N}\int_{\mathbb{R}^N} \frac{|u(x+y)-u(x)|^2}{|y|^{N+2s}}\,dx\,dy = C_s \int_{\mathbb{R}^N} |\xi|^{2s} |\hat{u}(\xi)|^2\,d\xi$$

Par conséquent

$$\operatorname{Min}\{1, C_s\} \|u\|_{\dot{H}^s(\mathbb{R}^N)} \leq \|\tilde{u}\|_{\dot{H}^s(\mathbb{R}^N)} \leq \operatorname{Max}\{1, C_s\} \|u\|_{\dot{H}^s(\mathbb{R}^N)}$$

Ce qui achève la demonstration